Adaptive Macro-Micro Control of Nonlinear Underwater Robotic Systems

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Abstract - **Adaptive macro-micro control of nonlinear underwater vehicle-manipulator systems is addressed in this paper. The adaptive passivity-based control scheme is formulated in an augmented task-space where both the underwater vehicle and the end-effector have 6 degrees of freedom (DOF). The underwater vehicle represents a slow gross positioning part while the manipulator's end-effector represents the fast part. Underwater vehicle-manipulator systems where the number of inputs exceeds the number of controllable degrees of freedom (DOF) are also considered. The input uncertainty in such systems is considered in detail. Global stability is ensured by applying Barbalat's Lyapunov-like lemma for non-autonomous systems.**

I. INTRODUCTION

Non-destructive testing of underwater structures requires high performance manoeuvres of underwater vehicle-manipulator systems within and close to underwater installations. This imposes stricter requirements on the control system, particularly when macro-micro control is of interest i.e. coordination of the motion between the underwater vehicle and a manipulator arm. The scheme presented in this paper is intended for the macro-micro control of such systems.

In robotics adaptive passivity-based control has given high performance for robotic systems in the form: $\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q})=\boldsymbol{\tau}$, e.g. Slotine and Li [11], Sadegh and Horowitz [10] and Kelly and Carelli [9]. For the underwater vehicle-manipulator systems it is desirable to consider a more general model class. In particular uncertainties in the input matrix due to estimated thruster characteristics may cause problems when applying standard control schemes.

Egeland and Sagli [3] suggest using the augmented task-space approach for feedback linearization of spacecraft-manipulator systems. For underwater vehicle-manipulator systems poorly known hydrodynamical coefficients are included among the rigid body parameters of the dynamical model. When designing controllers for high performance underwater robotic systems it is necessary to compensate in the model for parametric uncertainties, nonlinear thruster characteristics, nonlinear dynamics, nonlinear kinematics, nonlinearities due to hysteresis and actuator dead-zones. A conventional linear controller cannot guarantee high performance for such systems. This suggests a robust adaptive control design. This paper proposes a globally stable adaptive controller for underwater vehicle-manipulator systems based on the scheme presented by Slotine and Li [11].

The paper is outlined as follows: Section II describes the mathematical modelling of underwater robotic systems. Passivity based adaptive control and hybrid adaptive control applied to underwater robotic systems are discussed in Section III and IV. Some conclusions are given at the end of the paper.

II. MATHEMATICAL MODELLING OF UNDERWATER ROBOTIC SYSTEMS

The underwater vehicle-manipulator system may be treated as a macro-micro manipulator system where the manipulator gives fast and accurate end-effector motion and the underwater vehicle is the slower positioning part, Egeland [2].

A. Underwater Vehicle Equations of Motion

- **Kinematics:**

$$\dot{\boldsymbol{x}}_1=\boldsymbol{T}(\phi,\theta,\psi)\dot{\boldsymbol{q}}_1$$

where $\boldsymbol{T}(\phi,\theta,\psi)$ is an $m_1\times n_1$ block diagonal transformation matrix describing the vehicle's linear and angular velocities in local coordinates relative to an earth-fixed reference frame, see e.g. Fossen and Balchen [4]. The

Euler angles ϕ, θ and ψ correspond to roll, pitch and yaw. It is convenient to define the state vectors according to the SNAME notation i.e.: $\dot{\boldsymbol{q}}_1 = (u, v, w, p, q, r)^T$ where $\boldsymbol{q}$ is a virtual vector and $\boldsymbol{x}_1 = (x, y, z, \phi, \theta, \psi)^T$.

- **Vehicle dynamics:**

For underwater vehicles it is convenient to write the equations of motion as in Fossen [6] i.e.

$$\boldsymbol{M}_{rov}\ddot{\boldsymbol{q}}_1 + \boldsymbol{C}_{rov}(\dot{\boldsymbol{q}}_1)\,\dot{\boldsymbol{q}}_1 + \boldsymbol{D}_{rov}(\dot{\boldsymbol{q}}_1)\,\dot{\boldsymbol{q}}_1 + \boldsymbol{g}_{rov}(\boldsymbol{x}_1) = \boldsymbol{\tau}_1$$
$$\boldsymbol{\tau}_1 = \boldsymbol{B}_{rov}(\dot{\boldsymbol{q}}_1)\boldsymbol{u}_1$$

where $\boldsymbol{q}_1 \in \Re^{n_1}$ and $\boldsymbol{x}_1 \in \Re^{m_1}$. $\boldsymbol{\tau}_1 \in \Re^{n_1}$ is the control force and moment vector while $\boldsymbol{u}_1 \in \Re^{p_1}$ is a vector of propeller square angular velocities. $\boldsymbol{B}_{rov}$ is an $n_1 \times p_1$ velocity dependent thruster configuration matrix. The thruster configuration matrix may be obtained from open water tests. Open water thruster characteristics are discussed more closely in Fossen and Sagatun [7]. $\boldsymbol{M}_{rov}$ is an $n_1 \times n_1$ inertia matrix including hydrodynamic added mass, $\boldsymbol{C}_{rov}$ is an $n_1 \times n_1$ matrix of centripetal and Coriolis terms, $\boldsymbol{D}_{rov}$ is an $n_1 \times n_1$ matrix of damping terms and $\boldsymbol{g}_{rov}(\boldsymbol{x}_1)$ is an $n_1 \times 1$ vector of hydrostatic forces and moments. All these terms are described more closely in Fossen [6].

B. Robot manipulator

The dynamics and kinematics of an n_2 -link rigid robot manipulator when friction is neglected can be written in a similar manner as

- **Kinematics:**

$$\boldsymbol{x}_2 = \boldsymbol{h}(\boldsymbol{x}_1, \boldsymbol{q}_2) \tag{1}$$

where $\boldsymbol{x}_2 \in \Re^{m_2}$ is the task space coordinates and $\boldsymbol{q}_2 \in \Re^{n_2}$ is a vector of joint displacements. $\boldsymbol{h}(\cdot)$ is a mapping from the space spanned by $(\boldsymbol{x}_1, \boldsymbol{q}_2)$ down to the $\boldsymbol{x}_2$ space.

- **Manipulator dynamics:**

$$\boldsymbol{M}_{man}(\boldsymbol{q}_2)\ddot{\boldsymbol{q}}_2 + \boldsymbol{C}_{man}(\boldsymbol{q}_2, \dot{\boldsymbol{q}}_2)\,\dot{\boldsymbol{q}}_2 + \boldsymbol{g}_{man}(\boldsymbol{q}_2) = \boldsymbol{\tau}_2$$

where $\boldsymbol{\tau}_2 \in \Re^{n_2}$ is a vector of applied joint torques or forces, $\boldsymbol{M}_{man}$ is an $n_2 \times n_2$ inertia matrix, $\boldsymbol{C}_{man}(\dot{\boldsymbol{q}}_2, \boldsymbol{x}_2)$ is an $n_2 \times n_2$ matrix of centripetal and Coriolis torques and $\boldsymbol{g}_{man}(\boldsymbol{x}_2)$ is an $n_2 \times 1$ vector of hydrostatic forces.

C. Underwater Vehicle and Manipulator

Combinations of parallel and serial mechanical structures, e.g. macro-micro manipulators, are analyzed in Khatib [8]. Let the generalized coordinates and control input be defined as

$$\boldsymbol{q}^T = (\boldsymbol{q}_1^T, \boldsymbol{q}_2^T) \quad , \quad \boldsymbol{x}^T = (\boldsymbol{x}_1^T, \boldsymbol{x}_2^T) \quad , \quad \boldsymbol{u}^T = (\boldsymbol{u}_1^T, \boldsymbol{u}_2^T)$$

where $\boldsymbol{q} \in \Re^n$, $\boldsymbol{x} \in \Re^m$ and $\boldsymbol{u} \in \Re^p$. Hence, the dynamic and kinematic equations of the underwater vehicle-manipulator system can be expressed in a compact form as

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{D}(\dot{\boldsymbol{q}})\,\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{q}) = \boldsymbol{B}(\dot{\boldsymbol{q}})\boldsymbol{u}$$
$$\dot{\boldsymbol{x}} = \boldsymbol{J}(\boldsymbol{x}, \boldsymbol{q})\dot{\boldsymbol{q}} \tag{2}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ are complicated functions of the elements in the matrices $\boldsymbol{M}_{rov}$, $\boldsymbol{M}_{man}$, $\boldsymbol{C}_{rov}$, $\boldsymbol{C}_{man}$ and $\boldsymbol{D}_{rov}$ while

$$\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{q}) = \begin{bmatrix} \boldsymbol{g}_{rov} + \boldsymbol{g}_{man} \\ \boldsymbol{g}_{man} \end{bmatrix} \quad , \quad \boldsymbol{B}(\dot{\boldsymbol{q}}) = \begin{bmatrix} \boldsymbol{B}_{rov} & 0 \\ 0 & \boldsymbol{I} \end{bmatrix}$$

For the underwater vehicle-manipulator systems $\boldsymbol{J}$ may be interpreted as a generalized Jacobian where

$$\boldsymbol{J}(\boldsymbol{x}, \boldsymbol{q}) = \begin{bmatrix} \boldsymbol{J}_{11} & \boldsymbol{J}_{12} \\ \boldsymbol{J}_{21} & \boldsymbol{J}_{22} \end{bmatrix} = \begin{bmatrix} \boldsymbol{T} & 0 \\ \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{x}_1}\boldsymbol{T} & \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{q}_2} \end{bmatrix}$$

Hence, $\boldsymbol{J}_{22}$ can be interpreted as the manipulator Jacobian. The off-diagonal matrix $\boldsymbol{J}_{21}$ describes the influence of the vehicle's motion on the manipulator motion.

III. ADAPTIVE CONTROL DESIGN

B known and p $\geq$ n

We will restrict our treatment to nonlinear systems with equal or more control inputs than controllable DOF i.e. $p \geq n$. Assume that the underwater vehicles earth-fixed position and orientation vector $\boldsymbol{x}_1$ and the vehicle-fixed linear and angular velocity vector $\dot{\boldsymbol{q}}_1$ are measured. Also, assume that the manipulator joint displacements $\boldsymbol{q}_2$ and joint velocities $\dot{\boldsymbol{q}}_2$ are measured. Hence,

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{h}(\boldsymbol{x}_1, \boldsymbol{q}_2) \end{pmatrix} \qquad \dot{\boldsymbol{q}} = \begin{pmatrix} \dot{\boldsymbol{q}}_1 \\ \dot{\boldsymbol{q}}_2 \end{pmatrix}$$

Consider the underwater robotic system, Eq. 2, which can be expressed as

$$\boldsymbol{M}^*\ddot{\boldsymbol{x}} + \boldsymbol{C}^*\dot{\boldsymbol{x}} + \boldsymbol{D}^*\dot{\boldsymbol{x}} + \boldsymbol{g}^* = \boldsymbol{J}^{-T}\boldsymbol{B}\boldsymbol{u}$$

Here we assumed that $\boldsymbol{J}$ is non-singular and

$$\begin{aligned} \boldsymbol{M}^*(\boldsymbol{x}, \boldsymbol{q}) &= \boldsymbol{J}^{-T}\boldsymbol{M}\boldsymbol{J}^{-1} \\ \boldsymbol{C}^*(\boldsymbol{x}, \boldsymbol{q}, \dot{\boldsymbol{q}}) &= \boldsymbol{J}^{-T}\left[\boldsymbol{C} - \boldsymbol{M}\boldsymbol{J}^{-1}\dot{\boldsymbol{J}}\right]\boldsymbol{J}^{-1} \\ \boldsymbol{D}^*(\boldsymbol{x}, \boldsymbol{q}, \dot{\boldsymbol{q}}) &= \boldsymbol{J}^{-T}\boldsymbol{D}\boldsymbol{J}^{-1} \\ \boldsymbol{g}^*(\boldsymbol{x}, \boldsymbol{q}) &= \boldsymbol{J}^{-T}\,\boldsymbol{g} \end{aligned}$$

Assume the desired trajectory ($\ddot{\boldsymbol{x}}_d$, $\dot{\boldsymbol{x}}_d$ and $\boldsymbol{x}_d$) to be bounded. Let $\tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_d$ be the tracking error. Slotine and Li [11] suggest defining a measure of tracking $\boldsymbol{s}$ such that

$$\boldsymbol{s} = \dot{\tilde{\boldsymbol{x}}} + \lambda \tilde{\boldsymbol{x}} \tag{3}$$

where λ is a strictly positive constant which may be interpreted as the control bandwidth. It is convenient to rewrite Eq. 3 as

$$\boldsymbol{s} = \dot{\boldsymbol{x}} - \dot{\boldsymbol{x}}_r \quad \text{where} \quad \dot{\boldsymbol{x}}_r = \dot{\boldsymbol{x}}_d - \lambda \tilde{\boldsymbol{x}}$$

Let $\hat{\boldsymbol{\theta}}$ be the time-varying parameter vector estimate and let $\tilde{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}} - \boldsymbol{\theta}$ be the parameter error vector. To prove global stability Slotine and Benedetto [12] suggest using a Lyapunov-like function candidate

$$V(\boldsymbol{s}, \tilde{\boldsymbol{\theta}}, t) = \frac{1}{2}\boldsymbol{s}^T \boldsymbol{M}^* \boldsymbol{s} + \frac{1}{2}\tilde{\boldsymbol{\theta}}^T \boldsymbol{\Gamma} \tilde{\boldsymbol{\theta}} \;, \quad \boldsymbol{M}^* = \boldsymbol{M}^{*T} > 0$$

where $\boldsymbol{\Gamma}$ is a symmetric positive definite weighting matrix of appropriate dimension. Differentiating V with respect to time and using the skew symmetry property $\dot{\boldsymbol{x}}^T(\dot{\boldsymbol{M}}^* - 2\boldsymbol{C}^*)\dot{\boldsymbol{x}} = 0$ yields

$$\begin{aligned} \dot{V} &= -\boldsymbol{s}^T \boldsymbol{D}^* \boldsymbol{s} + \dot{\tilde{\boldsymbol{\theta}}}^T \boldsymbol{\Gamma} \tilde{\boldsymbol{\theta}} \\ &+ \boldsymbol{s}^T(\boldsymbol{B}^* \boldsymbol{u} - \boldsymbol{M}^* \ddot{\boldsymbol{x}}_r - \boldsymbol{C}^* \dot{\boldsymbol{x}}_r - \boldsymbol{D}^* \dot{\boldsymbol{x}}_r - \boldsymbol{g}^*) \end{aligned}$$

Fossen [5] suggests defining a virtual reference vector $\dot{q}_r$ satisfying the transformation

$$\dot{\boldsymbol{x}}_r = \boldsymbol{J}(\boldsymbol{x}, q)\, \dot{q}_r$$

Hence, the virtual reference vectors $\dot{q}_r$ and $\ddot{q}_r$ can be calculated as

$$\begin{aligned} \dot{q}_r &= \boldsymbol{J}^{-1}(\boldsymbol{x}, q)\, \dot{\boldsymbol{x}}_r \\ \ddot{q}_r &= \boldsymbol{J}^{-1}(\boldsymbol{x}, q)\ddot{\boldsymbol{x}}_r - \boldsymbol{J}^{-1}(\boldsymbol{x}, q)\dot{\boldsymbol{J}}(\boldsymbol{x}, q)\boldsymbol{J}^{-1}(\boldsymbol{x}, q)\, \dot{\boldsymbol{x}}_r \end{aligned}$$

Hence, $\dot{V}$ can be written as

$$\begin{aligned} \dot{V} &= -\boldsymbol{s}^T \boldsymbol{D}^* \boldsymbol{s} + \dot{\tilde{\boldsymbol{\theta}}}^T \boldsymbol{\Gamma} \tilde{\boldsymbol{\theta}} \\ &+ \boldsymbol{s}^T \boldsymbol{J}^{-T}(\boldsymbol{B}\boldsymbol{u} - \boldsymbol{M}\ddot{q}_r - \boldsymbol{C}\dot{q}_r - \boldsymbol{D}\dot{q}_r - g) \end{aligned} \tag{4}$$

Let the control law be chosen as

$$\boldsymbol{u} = \boldsymbol{B}^\dagger \left[\hat{\boldsymbol{M}}\ddot{q}_r + \hat{\boldsymbol{C}}\dot{q}_r + \hat{\boldsymbol{D}}\dot{q}_r + \hat{g} - \boldsymbol{J}^T \boldsymbol{K}_D \boldsymbol{s}\right] \tag{5}$$

where the hat denotes the adaptive estimates, $\boldsymbol{K}_D$ is a positive definite design matrix of appropriate dimensions and $\boldsymbol{B}^\dagger$ is a generalized inverse of the known matrix $\boldsymbol{B}$ found by minimizing an energy cost function

$$\text{Min } J = \frac{1}{2}\boldsymbol{u}^T \boldsymbol{W} \boldsymbol{u} \quad \text{subject to} \quad \boldsymbol{\tau} = \boldsymbol{B}\boldsymbol{u}$$

where $\boldsymbol{W}$ is positive definite matrix. The solution is well known and can be written as

$$\boldsymbol{B}^\dagger = \boldsymbol{W}^{-1}\boldsymbol{B}^T(\boldsymbol{B}\boldsymbol{W}^{-1}\boldsymbol{B}^T)^{-1}$$

For underwater vehicle-manipulator systems, the elements in $\boldsymbol{W}$ should be selected such that the use of manipulator torques are much more inexpensive than the use of thrusters i.e. providing a means of saving battery energy. The parameter update law is derived by substituting Eq. 5 into the expression for $\dot{V}$ which yields

$$\begin{aligned} \dot{V} &= -\boldsymbol{s}^T(\boldsymbol{K}_D + \boldsymbol{D}^*)\boldsymbol{s} + \dot{\tilde{\boldsymbol{\theta}}}^T \boldsymbol{\Gamma} \tilde{\boldsymbol{\theta}} \\ &\quad + \boldsymbol{s}^T \boldsymbol{J}^{-T}(\tilde{\boldsymbol{M}}\ddot{q}_r + \tilde{\boldsymbol{C}}\dot{q}_r + \tilde{\boldsymbol{D}}\dot{q}_r + \tilde{g}) \end{aligned}$$

where $\tilde{\boldsymbol{M}} = \hat{\boldsymbol{M}} - \boldsymbol{M}$, $\tilde{\boldsymbol{C}} = \hat{\boldsymbol{C}} - \boldsymbol{C}$, $\tilde{\boldsymbol{D}} = \hat{\boldsymbol{D}} - \boldsymbol{D}$ and $\tilde{g} = \hat{g} - g$. Using the parameterization of Fossen [5], namely

$$\boldsymbol{M}\ddot{q}_r + \boldsymbol{C}\dot{q}_r + \boldsymbol{D}\dot{q}_r + g = \boldsymbol{\Phi}(\boldsymbol{x}, \dot{q}, \dot{q}_r, \ddot{q}_r)\, \theta$$

where $\boldsymbol{\Phi}(\boldsymbol{x}, \dot{q}, \dot{q}_r, \ddot{q}_r)$ is a known regressor matrix of appropriate dimensions, the expression for $\dot{V}$ can be written as

$$\dot{V} = -\boldsymbol{s}^T(\boldsymbol{K}_D + \boldsymbol{D}^*)\boldsymbol{s} + \tilde{\boldsymbol{\theta}}^T(\boldsymbol{\Gamma}\dot{\tilde{\boldsymbol{\theta}}} + \boldsymbol{\Phi}^T \boldsymbol{J}^{-1}\boldsymbol{s})$$

This suggests that the parameter adaption law should be chosen as

$$\dot{\hat{\theta}} = -\boldsymbol{\Gamma}^{-1}\boldsymbol{\Phi}^T(\boldsymbol{x}, \dot{q}, \dot{q}_r, \ddot{q}_r)\boldsymbol{J}^{-1}(\boldsymbol{x}, q)\, \boldsymbol{s}$$

Assuming $\dot{\theta} = 0$, finally yields

$$\dot{V} = -\boldsymbol{s}^T(\boldsymbol{K}_D + \boldsymbol{D}^*)\boldsymbol{s} \leq 0$$

Applying Barbalat's lemma shows that the output error converges to the surface $\boldsymbol{s} = 0$ and thus $\tilde{\boldsymbol{x}} \to 0$. Notice, that the adaptive scheme does not require the knowledge of the manipulator's inverse kinematics.

IV. HYBRID ADAPTIVE CONTROL

B unknown and p $\geq$ n

Sliding control has been applied to underwater vehicles by Yoerger and Slotine [14] and Cristi et al. [1]. In this section we will derive a hybrid (adaptive and sliding) controller which compensates for uncertainties in the $\boldsymbol{B}$ matrix by adding a discontinuous term to the adaptive control law. Assume that the thruster configuration matrix $\boldsymbol{B}$ satisfies a multiplicative uncertainty

$$B(\dot{q}) = (I + \Delta) B_o(\dot{q}) \quad , \quad |\Delta_{ij}| \leq U_{ij} \qquad (6)$$

where ($i = 1..n$, $j = 1..n$) and $\overline{\sigma}(\Delta) < 1$. Substituting this expression into Eq. 4, yields

$$\dot{V} = -s^T D^* s + \dot{\tilde{\theta}}^T \Gamma \tilde{\theta} + s^T J^{-T} \left[(I + \Delta) B_o u - \Phi \, \theta \right]$$

Taking the control law to be

$$u = B_o^\dagger \left[\Phi \, \hat{\theta} - J^T K_D s - k \,.\!\times\, sgn(J^{-1} s) \right]$$

where we have included a switching term $k \,.\!\times\, sgn(J^{-1}s)$ to compensate for the uncertainty in the B matrix. The operator $.\!\times$ denotes the Schur product (element by element multiplication) while sgn simply is the signum function. Conditions on the the switching gain vector k is found by requiring that

$$\dot{V} = -s^T (K_D + D^*) s + (J^{-1} s)^T \left[\Delta (\Phi \hat{\theta} - J^T K_D s) - (I + \Delta) \, k \,.\!\times\, sgn(J^{-1} s) \right] \leq 0$$

Indeed, this is satisfied if the switching gain vector k is chosen such that $k_i \geq k'_i$ (i=1...n), where k' satisfies

$$(I - \overline{U}) \, k' = U \, |\Phi \hat{\theta} - J^T K_D s| + \eta \quad , \quad \eta_i > 0 \quad (7)$$

where the elements U_{ij} are defined in Eq. 6 and the matrix $\overline{U}$ is defined as

$$\overline{U} = \begin{bmatrix} U_{11} & -U_{12} & \cdots & -U_{1n} \\ -U_{21} & U_{22} & & -U_{2n} \\ \vdots & & \ddots & \vdots \\ -U_{n1} & -U_{n2} & \cdots & U_{nn} \end{bmatrix}$$

Hence,

$$\dot{V} \leq -s^T (K_D + D^*) s - \eta^T |J^{-1} s| \leq 0$$

Applying Barbalat's lemma implies that $s \rightarrow 0$ and $\tilde{x} \rightarrow 0$. The matrix K_D directly accelerates the convergence. Slotine suggests to smooth out the control law within boundary layers to avoid chattering, which is imposed by the discontinuous term $k \,.\!\times\, sgn(J^{-1}s)$. This is described more closely in Slotine and Li [13].

CONCLUSIONS

An adaptive controller for nonlinear underwater robotic systems have been presented in this paper. The cases where the control input matrix B is both known and unknown have been investigated. An energy cost function, weighting both the thruster forces and the manipulators torques, has been applied to avoid excessive use of the control energy. Macro-micro control of an underwater vehicle-manipulator system is used through this paper to illustrate the control scheme.

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