

High Performance Ship Autopilot With Wave Filter

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Abstract

This paper describes the application of Lyapunov stability theory to automatic steering of ships. The motivation for using nonlinear control theory is that nonlinearities can be compensated for in a systematic manner. For instance, this allows the designer to design the autopilot system for a ship described by the nonlinear models of Norrbin (1963) and Bech and Wagner Smith (1969) both utilizing a nonlinear maneuvering characteristic. In fact, nonlinear maneuvering properties are observed for most large tankers. Furthermore, it is emphasized on showing how the controller gains of the nonlinear autopilot can be tuned in accordance to theoretical values obtained from linear quadratic (LQ) optimal control theory. Nonlinear control system design in the context of parameter estimation and wave filtering is also discussed. The advantages of the optimal autopilot is improved performance and reduced fuel consumption.

Finally, some experimental results with the Lyapunov based autopilot in combination with a parameter estimation algorithm are reported. These experiments were performed for a cargo ship in weather conditions corresponding to sea state code 2, that is smooth sea. As a consequence of this, simulation results are used exclusively to demonstrate the state estimator and wave filter design for higher sea state codes.

1 Introduction

Since the rapid increase in oil prices in 1973, advanced optimal and adaptive autopilots have been designed to reduce the fuel consumption and to improve the performance of the ship autopilot, see e.g. Van Amerongen and Van Nauta Lemke (1978, 1980), Van Amerongen (1984), Holzhüter and Strauch (1987), Koyama (1967), Källström (1979), Källström, Åström, Thorell, Eriksson and Sten (1979), Källström and Theorén (1992), Källström and Åström (1981), Ohtsu, Horigome and Kitagawa (1979), Sælid and Jenssen (1983) and Sælid, Svanes, Onshus and Jenssen (1984). Most of the work reported in the literature is based on linear control theory. In this paper it is shown how nonlinear control theory can be applied in the design of a high performance autopilot for ships with nonlinear maneuvering characteristics. Extensions to a fully nonlinear adaptive autopilot is discussed by Fossen and Paulsen (1992).

Finally, an observer based pole-placement algorithm intended for adaptive wave filtering is presented. The advantages of the new wave filter algorithm to the standard Kalman filter algorithm is that the tuning of the estimator gain is simplified. In fact, it is straightforward to incorporate

the effect of changing weather conditions and time-varying ship and wave model parameters in the design.

2 Ship Steering Dynamics

We will in this section briefly review the most standard models for ship steering.

2.1 Linear Models

The ship steering dynamics is usually described by one of the following Nomoto models:

$$\begin{aligned} \text{Nomoto 1st-order:} \quad & T \dot{r} + r = K \delta \\ \text{Nomoto 2nd-order:} \quad & T_1 T_2 \ddot{r} + (T_1 + T_2) \dot{r} + r = K(\delta + T_3 \dot{\delta}) \end{aligned} \quad (1)$$

where r (deg/s) is the yaw rate and δ (deg) is the rudder angle. K is the Nomoto gain constant while the time constants T_i ($i = 1..3$) can be related to T (equivalent time constant) by:

$$T = T_1 + T_2 - T_3 \quad (2)$$

The yaw angle is obtained from $\dot{\psi} = r$.

2.2 Linear Models With Added Nonlinearities

Norrbin (1963) and Bech and Wagner Smith (1969) propose to replace the linear term r with a nonlinear maneuvering characteristic $H_N(r)$ and $H_B(r)$ in Nomoto's 1st- and 2nd-order models, respectively. These models are written:

$$T \dot{r} + H_N(r) = K \delta \quad (3)$$

$$T_1 T_2 \ddot{r} + (T_1 + T_2) \dot{r} + K H_B(r) = K(\delta + T_3 \dot{\delta}) \quad (4)$$

The functions $H_N(r)$ and $H_B(r)$ are used to describe the nonlinear maneuvering characteristic produced by Bech's reverse spiral maneuver. The maneuvering characteristic is usually taken to be a third order polynomial:

$$H_N(r) = n_3 r^3 + n_2 r^2 + n_1 r + n_o \quad (5)$$

$$H_B(r) = b_3 r^3 + b_2 r^2 + b_1 r + b_o \quad (6)$$

For a course-unstable ship we will have that $b_1 < 0$ while a course stable ship satisfies $b_1 > 0$. A single screw propeller or asymmetry in the hull will cause a non-zero value of b_o . Similarly, symmetry in the hull implies that $b_2 = 0$. Since a constant rudder angle is required to compensate for constant steady state wind and current disturbances, the bias term b_o could conveniently be treated as an additional rudder off-set. This in turn implies that a large number of ships can be described by the polynomial:

$$H_B(r) = b_3 r^3 + b_1 r \quad (7)$$

The coefficients b_i ($i=0..3$) are related to those of Norrbin's model n_i ($i=0..3$) by:

$$n_i = \frac{b_i}{|b_1|} \quad (8)$$

Hence, $n_1 = 1$ for a course-stable ship and $n_1 = -1$ for a course-unstable ship. A more detailed discussion on ship steering dynamics is found in Fossen and Yoerger (1994).

3 Nonlinear Autopilot Design

In this section, we will show how nonlinear stability theory can be used to design a nonlinear autopilot.

3.1 Control Objective

The control objective is to track the desired states $\psi_d(t)$, $r_d(t)$ and $\dot{r}_d(t)$. During course-keeping, the desired states are defined by:

$$\psi_d(t) = \text{constant}; \quad \dot{r}_d(t) = r_d(t) = 0 \quad (9)$$

During turning of the ship we suggest to specify the desired yaw angle $\psi_d(t)$ by a 2nd-order reference model:

$$\ddot{\psi}_d(t) + 2\zeta_m\omega_m\dot{\psi}_d(t) + \omega_m^2\psi_d(t) = \omega_m^2\psi_r \quad (10)$$

where ψ_r is the commanded heading angle (steady-state), ζ_m is the desired damping ratio of the closed-loop system, typically chosen in the interval:

$$0.8 \leq \zeta_m \leq 1.0 \quad (11)$$

and ω_m is the desired natural frequency.

3.2 Nonlinear Control Design

For notational simplicity, we will consider Norrbin's steering equations of motion in the following form:

$$m\dot{r} + d(r)r = \delta \quad (12)$$

where $m = T/K$ and:

$$d(r) = (n_3 r^2 + n_2 r + n_1)/K \quad (13)$$

The constant n_o is omitted since this parameter can be treated as an additional rudder off-set to be compensated for by the controller integral action. In addition to this, slowly-varying disturbances (wind, wave drift forces and currents) are assumed included in the parameter n_o . Let the *pseudo-kinetic energy* of the ship be described by the function:

$$V(s) = \frac{1}{2}m s^2 \quad (14)$$

where s is a measure of tracking defined according to, Slotine and Li (1987):

$$s = \tilde{r} + 2\lambda \tilde{\psi} + \lambda^2 \int_0^t \tilde{\psi}(\tau) d\tau \quad (15)$$

Here $\tilde{\psi} = \psi - \psi_d$ and $\tilde{r} = r - r_d$ are the yaw angle and yaw rate tracking errors, respectively. $\lambda > 0$ can be interpreted as the control bandwidth. Let us define a *virtual yaw rate* v as:

$$v = r - s = r_d - 2\lambda \tilde{\psi} - \lambda^2 \int_0^t \tilde{\psi}(\tau) d\tau \quad (16)$$

Differentiating V w.r.t. time, yields:

$$\dot{V} = m s \dot{s} = s [m \dot{r} - m \dot{v}] \quad (17)$$

Substitution of (12) into this expression for \dot{V} , yields:

$$\dot{V} = s [\delta - d(r) r - m \dot{v}] = -d(r) s^2 + s [\delta - m \dot{v} - d(r) v] \quad (18)$$

This suggests that the nonlinear control law could be chosen according to:

$$\boxed{\delta = m \dot{v} + d(r) v - K_d s} \quad (19)$$

where $K_d > 0$ is a design parameter. This yields:

$$\dot{V} = -[d(r) + K_d] s^2 \quad (20)$$

K_d must be chosen such that $\dot{V} \leq 0 \forall r$. A guideline could be to choose:

$$K_d = -d(r) + \frac{\lambda}{2} m > 0 \quad (21)$$

which yields:

$$\dot{V} = -\frac{\lambda}{2} m s^2 \leq 0 \quad (22)$$

According to Barbălat's lemma (see Appendix A) convergence of $V(t)$ to zero and thus $s(t)$ to zero is guaranteed. In view of (15), $\psi(t) \rightarrow \psi_d(t)$ in finite time. The “bandwidth” of the controller can be specified in terms of λ . Substituting (14) into (22), yields:

$$\dot{V}(t) = -\lambda V(t) \quad \implies \quad V(t) = e^{-\lambda t} V(0) \quad (23)$$

Hence, it is seen that an initial error $s(0) \neq 0$ implies that $V(0) = m s^2(0)/2 > 0$. Furthermore, $V(t)$ will converge exponentially to zero if $\lambda > 0$. Hence, fast convergence of the s -dynamics to zero can simply be monitored by specifying the design parameter λ large enough.

3.3 Analogy to Feedback Linearization

The control law (19) with K_d defined in (21) can be rewritten according to:

$$\delta = m a + d(r) r \quad (24)$$

Here a is defined as:

$$a = \dot{r}_d - \frac{5}{2}\lambda \tilde{r} - 2\lambda^2 \tilde{\psi} - \frac{1}{2}\lambda^3 \int_0^t \tilde{\psi}(\tau) d\tau \quad (25)$$

From the theory of feedback linearization a can be interpreted as the *commanded acceleration*. The nonlinear term $d(r)r$ is included to cancel out the model nonlinearities. Moreover, combining (24) with (12) yields the following linear system:

$$\ddot{\psi} = a \quad (26)$$

This linear control problem can be solved in terms of the new control variable a . More generally, we could choose:

$$a = \dot{r}_d - K_d \tilde{r} - K_p \tilde{\psi} - K_i \int_0^t \tilde{\psi}(\tau) d\tau \quad (27)$$

where K_p , K_d and K_i can be interpreted as the proportional, derivative and integral gains, respectively. The first term in the commanded acceleration is simply an acceleration feedforward term. Finally, the nonlinear control law for δ is obtained by substitution of a into (24).

3.4 Nonlinear Optimal Control Design

A linear quadratic (LQ) optimal course-keeping controller ($\psi_d = \text{constant}$) for the system $\ddot{\psi} = a$ can be designed by minimizing the quadratic cost function:

$$\min J = \frac{1}{2} \int_0^T [(\psi_d - \psi)^2 + \lambda_1 r^2 + \lambda_2 a^2] d\tau \quad (28)$$

weighting the tracking error $\psi_d - \psi$ against the yawing rate r and the commanded acceleration a with weighting factors λ_1 and λ_2 , respectively. This yields the following steady-state solution for the optimal commanded acceleration:

$$a = K_p (\psi_d - \psi) - K_d r \quad (29)$$

where

$$K_p = \sqrt{\frac{1}{\lambda_2}} \quad K_d = \sqrt{2\sqrt{\frac{1}{\lambda_2}} + \frac{\lambda_1}{\lambda_2}} \quad (30)$$

Integral action may be obtained by modifying the commanded acceleration as:

$$a = K_p (\psi_d - \psi) - K_d r + K_i \int_0^T (\psi_d - \psi(\tau)) d\tau \quad (31)$$

where a suitable choice of K_i is:

$$K_i \approx \frac{K_p}{5 K_d} \quad (32)$$

This corresponds to: $T_i = 5 T_d$ in a PID controller.

3.5 Parameter Estimator

The model parameters are suggested estimated via standard recursive estimation techniques, for instance:

$$\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \mathbf{P}(k) \boldsymbol{\phi}(k-1) \mathbf{e}(k) \quad (33)$$

$$\mathbf{P}(k) = \frac{1}{\lambda} \left(\mathbf{P}(k-1) - \frac{\mathbf{P}(k-1) \boldsymbol{\phi}(k-1) \boldsymbol{\phi}^T(k-1) \mathbf{P}(k-1)}{\lambda + \boldsymbol{\phi}^T(k-1) \mathbf{P}(k-1) \boldsymbol{\phi}(k-1)} \right) \quad (34)$$

where $\boldsymbol{\theta}$, $\boldsymbol{\phi}$, \mathbf{e} and λ are different for different methods. A detailed discussion on recursive parameter estimation is found in Ljung and Söderström (1986).

4 Wave Filter Design

Wave filtering is concerned with the problem of removing the oscillatoric motion (1st-order wave disturbances) from the measurements. The 1st-order wave disturbances are usually around 0.1 Hz, which is close to the control bandwidth of the vessel. This is usually inside the bandwidth of the rudder servo of the ship. Nevertheless, we do not want the rudder to compensate for the oscillatory wave induced motion since this causes too much control action. The standard way of doing this is by applying a Kalman filter. Kalman filter based wave filtering has been discussed by numerous authors, for instance Balchen, Jenssen and Sælid (1976), Balchen, Jensen and Sælid (1980a), Balchen, Jensen, Mathisen and Sælid (1980b), Grimbly, Patton and Wise (1980a), Grimbly, Patton and Wise (1980b), Fung and Grimbly (1981), Fung and Grimbly (1983), Fotakis, Grimbly and Kouvaritakis (1982), Sælid and Jenssen (1983), Sælid, Jenssen and Balchen (1983), Holzhüter and Strauch (1987), Holzhüter (1992), Reid, Tugcu and Mears (1984) to mention some. The disadvantage with the Kalman filter based wave filter is that apriori information of the process and measurement noise covariance is required. Unfortunately, the process noise variance depends on the environmental disturbances. In addition to this the algebraic Ricatti equation (ARE) has to be solved for each sea state. In order to simplify this, we propose to use an observer utilizing a pole-placement algorithm. The structure of the observer is motivated by the steady-state Kalman filter.

4.1 Observer Structure

It is common to assume that the total motion of the ship-wave system can be described in terms of a low-frequency (LF) model representing the motion of the vessel and a high-frequency (HF) first-order wave induced motion. For a ship autopilot system, this assumption suggests that we can write the LF yaw dynamics as (Nomoto 1st-order):

$$\dot{\hat{\psi}}_L = \hat{r}_L + K_1 (\psi - \hat{\psi}_L - \hat{\psi}_H) \quad (35)$$

$$\dot{\hat{r}}_L = -\frac{1}{T} \hat{r}_L + \frac{K}{T} \delta + K_2 (\psi - \hat{\psi}_L - \hat{\psi}_H) \quad (36)$$

where ψ is yaw angle measurement and $\hat{\psi}_L$ and \hat{r}_L are the LF state estimates. K_1 and K_2 are two estimator gains to be interpreted later. The HF motion estimator is usually based on a linear approximation of the wave spectrum. The simplest solution uses a damped oscillator, that is:

$$\psi_H(s) = \frac{K_w s}{s^2 + 2\zeta\omega_n s + \omega_n^2} w(s) \quad (37)$$

where w is a zero mean Gaussian white noise sequence, ω_n is the dominating wave frequency and ζ is the damping ratio. This suggests that the HF state estimator could be written:

$$\dot{\hat{\xi}}_H = \hat{\psi}_H + K_3 (\psi - \hat{\psi}_L - \hat{\psi}_H) \quad (38)$$

$$\dot{\hat{\psi}}_H = -2\zeta\omega_n \hat{\psi}_H - \omega_n^2 \hat{\xi}_H + K_4 (\psi - \hat{\psi}_L - \hat{\psi}_H) \quad (39)$$

where K_3 and K_4 are the HF estimator gains. For simplicity (without loss of generality) we have assumed that the rudder off-set due to slowly-varying environmental disturbances can be neglected.

4.2 Pole-Placement Algorithm

Introducing the notation $\Delta(\cdot) = (\cdot) - (\hat{\cdot})$ implies that the estimation error dynamics can be written:

$$\begin{bmatrix} \Delta\dot{\psi}_L \\ \Delta\dot{r}_L \\ \Delta\dot{\xi}_H \\ \Delta\dot{\psi}_H \end{bmatrix} = \begin{bmatrix} -K_1 & 1 & 0 & -K_1 \\ -K_2 & -\frac{1}{T} & 0 & -K_2 \\ -K_3 & 0 & 0 & 1 - K_3 \\ -K_4 & 0 & -\omega_n^2 & -2\zeta\omega_n - K_4 \end{bmatrix} \begin{bmatrix} \Delta\psi_L \\ \Delta r_L \\ \Delta\xi_H \\ \Delta\psi_H \end{bmatrix} \quad (40)$$

Hence, the characteristic equation can be shown to satisfy:

$$\pi(s) = s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_o \quad (41)$$

where

$$a_3 = K_1 + K_4 + 2\zeta\omega_n + 1/T \quad (42)$$

$$a_2 = (1/T + 2\zeta\omega_n) K_1 + K_2 - \omega_n^2 K_3 + 1/T K_4 + (\omega_n^2 + 2\zeta\omega_n/T) \quad (43)$$

$$a_1 = (2\zeta\omega_n/T + \omega_n^2) K_1 + 2\zeta\omega_n K_2 - \omega_n^2/T K_3 + \omega_n^2/T \quad (44)$$

$$a_o = \omega_n^2/T K_1 + \omega_n^2 K_2 \quad (45)$$

The eigenvalue assignment can be done by requiring that the error dynamics must satisfy:

$$\prod_{i=1}^4 (s - p_i) \triangleq \pi(s) \quad (46)$$

where p_i ($i = 1..4$) are *real values* specifying the desired poles of the error dynamics. The solution can be written in abbreviated form as:

$$\mathbf{A} \mathbf{k} = \mathbf{b} \quad (47)$$

where $\mathbf{k} = (K_1, K_2, K_3, K_4)^T$ is the estimator gain vector and:

$$\mathbf{A} = \begin{bmatrix} \omega_n^2/T & \omega_n^2 & 0 & 0 \\ 2\zeta\omega_n/T + \omega_n^2 & 2\zeta\omega_n & -\omega_n^2/T & 0 \\ (1/T + 2\zeta\omega_n) & 1 & -\omega_n^2 & 1/T \\ 1 & 0 & 0 & 1 \end{bmatrix} \quad (48)$$

$$\mathbf{b} = \begin{bmatrix} p_1 p_2 p_3 p_4 \\ -p_1 p_2 p_4 - p_1 p_2 p_3 - p_2 p_3 p_4 - p_1 p_3 p_4 - \omega_n^2 / T \\ p_1 p_2 + p_2 p_4 + p_2 p_3 + p_1 p_4 + p_1 p_3 + p_3 p_4 - (\omega_n^2 + 2\zeta\omega_n / T) \\ -p_1 - p_2 - p_3 - p_4 - (2\zeta\omega_n + 1/T) \end{bmatrix} \quad (49)$$

Consequently, \mathbf{k} can be computed as:

$$\mathbf{k} = \mathbf{A}^{-1} \mathbf{b} \quad (50)$$

Notice that \mathbf{k} depends on the ship time constant (T) and the wave model parameters (ζ, ω_n) while p_i ($i = 1..4$) are four design parameters specifying the poles of the error dynamics. Typical Bode plots of the state estimates $\hat{\psi}_L$ and $\hat{\psi}_H$ are shown in Figure 1 and 2, respectively.

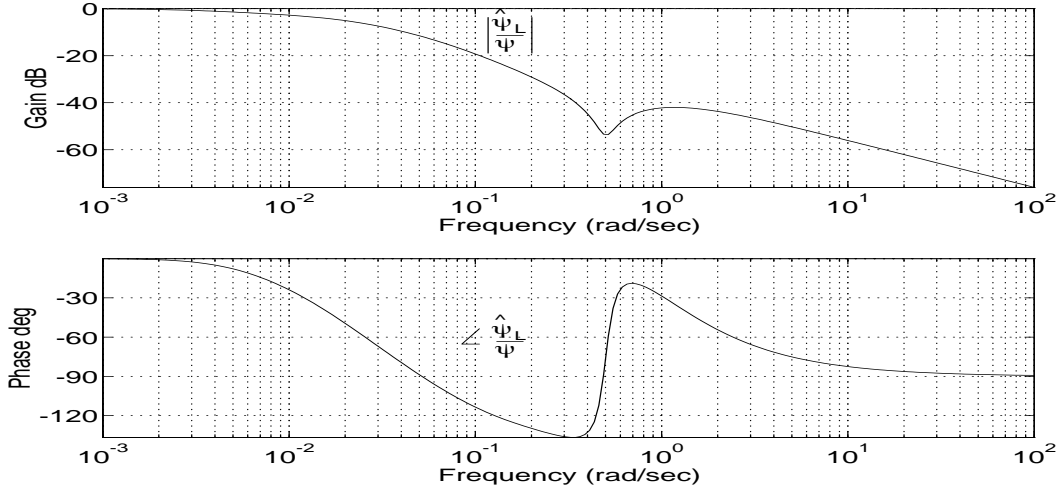


Figure 1: AFF diagram showing $\hat{\psi}_L(s)/\psi(s)$. Notice that wave disturbances are suppressed around the dominating wave (modal) frequency $\omega_n = 0.5$ (rad/s).

4.3 Tuning of the Wave Filter

Computer simulations shows that the observer is highly robust for parameter uncertainties if the pole locations are chosen carefully. A guideline could be to choose the real p_i -values according to:

$$p_1 < -1/T \quad (51)$$

$$p_2 < 0 \quad (52)$$

$$p_3 = p_4 < -\zeta\omega_n \quad (53)$$

Typical values are: $p_1 = -1.1/T$, $p_2 = -10^{-4}$ and $p_3 = p_4 = -15\zeta\omega_n$. The real part of the first two poles p_1 and p_2 are chosen slightly to the left of the open-loop poles $-1/T$ and 0 of the LF model. This ensures that the error dynamics of the LF states are faster than the ship dynamics. To obtain proper filtering the HF estimation error corresponding to the 1st-order wave disturbances should converge to zero much faster than the LF states. This is done by choosing $p_3 = p_4$ to the left of p_1 and p_2 . Both HF poles are real in order to avoid an oscillatory convergence of the HF

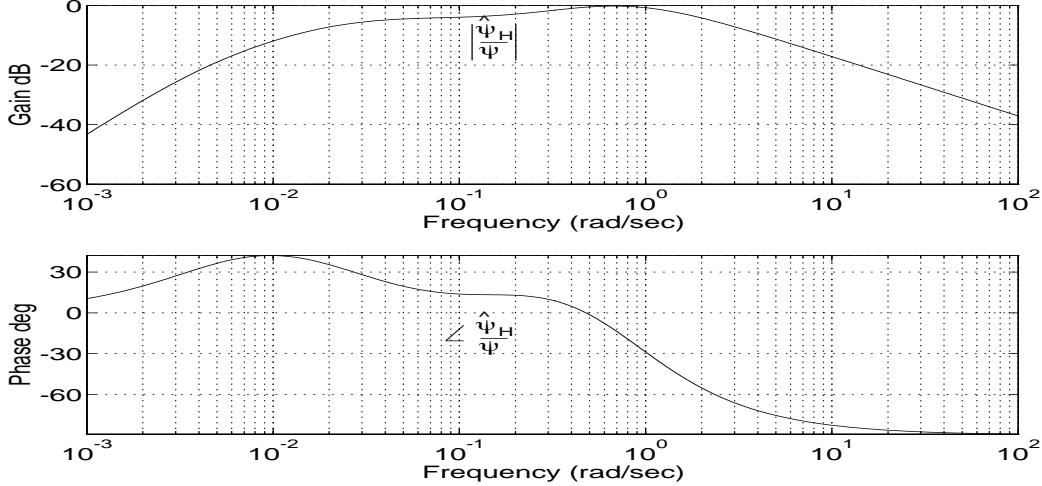


Figure 2: AFF diagram showing $\hat{\psi}_H(s)/\psi(s)$. Notice that the wave disturbance estimate nearly reflects the noisy heading measurement in the frequency band around the wave frequency $\omega_n = 0.5$ (rad/s) while LF components of the ship dynamics and HF noise are attenuated.

state estimation error to zero. Notice that the convergence of the HF state estimation error is not affected by the complex conjugate poles of the wave model.

The ship model parameters (K, T) can be found by applying a standard parameter estimator. Furthermore, the wave model damping ratio can be fixed to $\zeta = 0.01 - 0.1$ for most sea state codes while the wave frequency ω_n should be estimated on-line by applying a frequency tracker, see e.g. Sælid et al. (1983) or Holzhüter and Strauch (1987). If an on-line parameter estimator is used the observer gain vector \mathbf{k} can be computed on-line to obtain a fully adaptive wave filter.

5 Experimental and Simulation Results

5.1 Experimental Results With the M/S Nornews Express

An autopilot experiment was performed with the *M/S Nornews Express* which is cargo-ship operating on the west coast of Norway. The hull contour displacement is 4600 (*tons*) while the main dimensions are $L_{pp} = 110$ (*m*) and $B = 17.5$ (*m*). Unfortunately, this is a relatively small ship showing a highly linear behaviour. Consequently, it was decided to use a linear approximation of the maneuvering characteristic in the experiment. Moreover, this suggested that Nomoto's 1st-order model could be used. The Nomoto gain and time constant were estimated by applying an off-line recursive parameter estimation algorithm to a $5^\circ - 10^\circ$ zig-zag maneuver. This resulted in the following numerical values for the ship parameters:

$$K = 0.35 \text{ (s}^{-1}\text{)} \quad T = 29.0 \text{ (s)} \quad (54)$$

Furthermore, the commanded rudder was computed according to:

$$\delta = m \dot{v} + d v - K_d s \quad (55)$$

Here $d = 1/K = 2.86$ and $m = T/K = 82.86$. Velocity gain scheduling with respect to forward speed U and length of hull L_{pp} was obtained by introducing: $K = K'(U/L_{pp})$ and $T = T'(L_{pp}/U)$

where K' and T' are the non-dimensional Nomoto gain and time constant, respectively. The performance of the autopilot is shown in Figure 3.

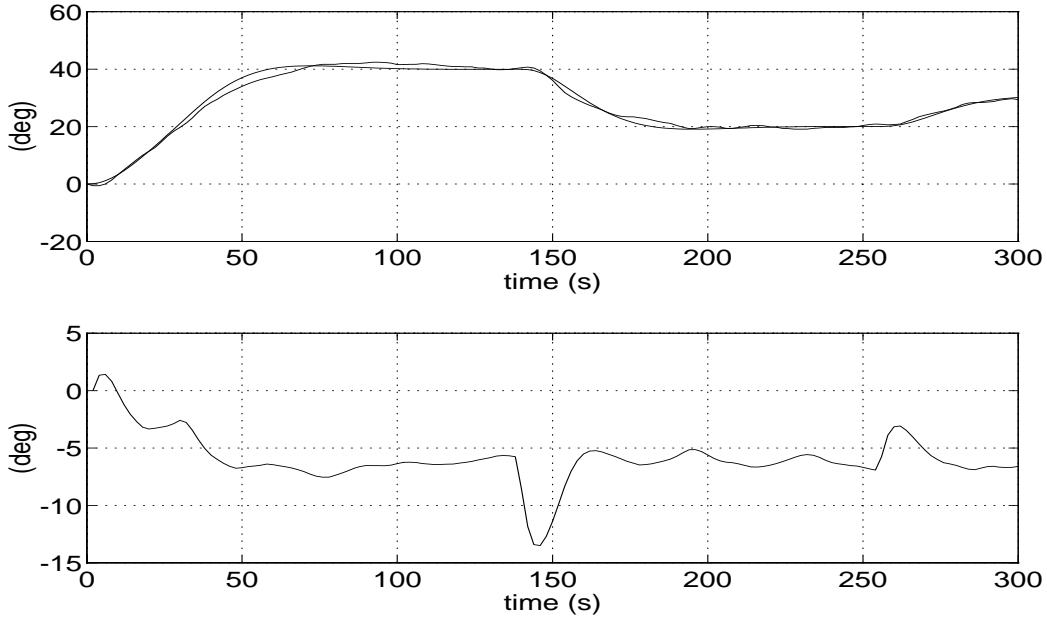


Figure 3: Course-changing maneuvers with the M/S Nornews Express. Desired and measured yaw angle versus time (upper plot) and rudder angle (lower plot) versus time.

As mentioned above, we did not have the opportunity to implement the proposed nonlinear control law during this first experiment since a larger ship probably is required. However, computer simulations with tankers show that performance improvements can be obtained by including a nonlinear maneuvering characteristic in (55). Hopefully, we will be able to verify this experimentally in the near future.

5.2 Simulation Study of the Observer Based Wave Filter in Bad Weather

The following computer simulations show that the wave filter gives excellent performance for high sea state codes. Again consider the cargo ship with $K = 0.35$ (s^{-1}) and $T = 29.0$ (s). Let $\omega_n = 0.5$ (rad/s), $\zeta = 0.1$ and

$$K_w = \begin{cases} 0.03 & \text{for } t \leq 100 \text{ (s)} \\ 0.10 & \text{for } t > 100 \text{ (s)} \end{cases}$$

In addition to this, we designed a simple PD-control law according to:

$$\delta = 0.83 (\psi_d - \hat{\psi}_L) - 13.71 \hat{r}_L \quad (56)$$

which yields a closed-loop bandwidth of $\omega_b = 0.1$ (rad/s). In the simulation study, the desired yaw angle was chosen as $\psi_d = 10^\circ$ for $t \leq 100$ (s) and $\psi_d = 0^\circ$ for $t > 100$ (s). Furthermore, the state estimates were computed by choosing the error dynamics poles according to: $p_1 = -1.1/T$, $p_2 = -10^{-4}$ and $p_3 = p_4 = -15 \zeta \omega_n$, which yields the estimator gain vector:

$$\mathbf{k} = (7.3528 \cdot 10^{-3}, -2.4501 \cdot 10^{-4}, -1.2689, 1.3962)^T \quad (57)$$

A typical time-series for this set of parameters is shown in Figure 4. We see that the state estimation errors are small and that excellent performance is obtained even for wave disturbances up to $\pm 10^\circ$ (deg) in amplitude.

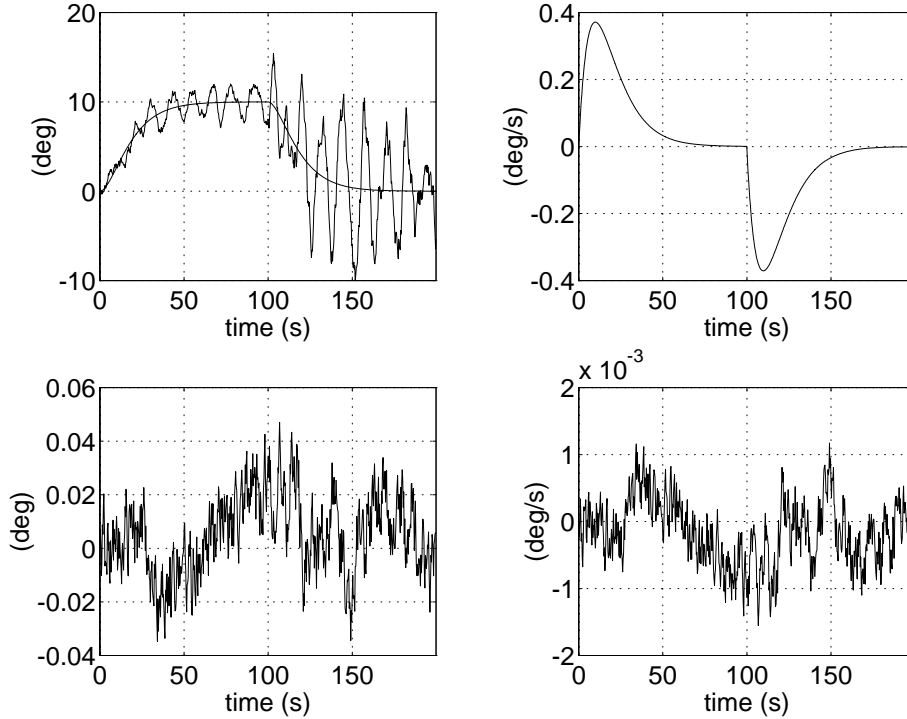


Figure 4: LF yaw angle ψ_L and measured yaw angle $\psi = \psi_L + \psi_H$ (upper left), LF yaw rate r_L (upper right), LF yaw angle estimation error $\Delta\psi_L$ (lower left) and LF yaw rate estimation error Δr_L (lower right) versus time. The simulation study was performed with a sampling time of 0.3 (s) while the yaw angle measurement noise was limited to ± 0.1 (deg).

6 Conclusions

A systematic approach for nonlinear control design with applications to ship steering has been discussed. The proposed control law has been successfully implemented and tested on a small cargo ship with excellent performance. Besides this a new observer intended for wave filtering is presented. The proposed wave filter algorithm is based on a pole-placement algorithm which is highly advantageous with respect to tuning and implementation. Simulation results are used to demonstrate the performance and robustness of the wave filter in bad weather. Future experiments with larger ships, which are highly nonlinear with respect to their maneuvering properties (Bech spiral), should be performed to investigate the performance of the nonlinear controller and the wave filter in bad weather.

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A Barbălat's Lemma

Lemma A.1 (Barbălat's Lemma for Global Convergence)

Assume that there exists a scalar function $V(\mathbf{x}, t)$ satisfying:

- $V(\mathbf{x}, t)$ is lower bounded
- $\dot{V}(\mathbf{x}, t)$ is negative semi-definite
- $\dot{V}(\mathbf{x}, t)$ is uniformly continuous in time

then $\dot{V}(\mathbf{x}, t) \rightarrow 0$ as $t \rightarrow \infty$.

Sufficient conditions for the first and last condition are:

Remark A.1 (Lower boundness) A sufficient condition for the scalar function $V(\mathbf{x}, t)$ to be lower bounded is that $V(\mathbf{x}, t)$ is positive semi-definite i.e. $V(\mathbf{x}, t) \geq 0 \quad \forall t \geq t_0$.

Remark A.2 (Uniform Continuity) A sufficient condition for a differentiable function $\dot{V}(\mathbf{x}, t)$ to be uniformly continuous is that its derivative $\ddot{V}(\mathbf{x}, t)$ is bounded $\forall t \geq t_0$.

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