NONLINEAR NON-MINIMUM PHASE RUDDER-ROLL DAMPING SYSTEM FOR SHIPS USING SLIDING MODE CONTROL

Trygve Lauvdal and Thor I. Fossen

Norwegian University of Science and Technology, Department of Engineering Cybernetics, N-7034 Trondheim, NORWAY. (E-mail: lauvdal@itk.unit.no, tif@itk.unit.no)

Abstract. A non-minimum phase nonlinear ship model from rudder angle to roll angle is used to design a stable controller for simultaneous roll damping and course keeping. The controller is stable in presence of 1st-order wave disturbance and modeling errors provided that the magnitude of the model errors and disturbances are known. Stability is proved by using Lyapunov theory. The simulation results show excellent performance and robustness.

Key Words. Nonlinear control, sliding mode control, marine system, rudder-roll stabilization, non-minimum phase systems.

1. INTRODUCTION

Roll damping and simultaneous heading control by means of rudders have been analyzed by numerous authors; Baitis et al. (1983, 1989), Blanke et al. (1989), Blanke and Christensen (1993), Katebi et al. (1987), Källström (1987), Källström et al. (1988), Källström and Schultz (1990), Van Amerongen et al. (1987), Van Amerongen and Van Nauta Lempke (1987), Van der Klugt (1987) and Zhou (1990). The common assumption in these papers is that the ship dynamics is linear and can be expressed as:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u \tag{1}$$

In this paper a nonlinear controller based on a nonlinear affine model is proposed. The ship dynamics is described by the following differential equation:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{b}u \tag{2}$$

where $\boldsymbol{f}(\boldsymbol{x})$ is a nonlinear function of the states. It is well-known that the dynamics in roll has non-minimum phase behaviour. This corresponds to zeros in the right half-plane for the linear model (1) and unstable zero dynamics for the nonlinear model (2) and implies limitations on the gain in the closed-loop system. In this paper this problem is solved by defining a sliding surface that gives the system stable zero dynamics if the sliding surface is used as output.

The resulting controller is a nonlinear non-minimum phase rudder-roll damping system for simultaneously roll and yaw control and it guarantees stability under a set of weak assumtions to be defined.

2. NONLINEAR SHIP MODEL

Consider the following SISO nonlinear ship model:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{f}^*(\boldsymbol{x}) + \boldsymbol{b}^*(\delta + w_H) \\ y &= \boldsymbol{c}^T\boldsymbol{x} \end{aligned} \tag{3}$$

where $\boldsymbol{x} \in \mathbb{R}^5$ is the state vector and defined as:

$$\boldsymbol{x} = [\, \phi \;\; p \;\; v \;\; r \;\; (\psi - \psi_d) \,]^T \tag{4}$$

Here ϕ is the roll angle, p is the angular velocity in roll, v is the sway velocity, r is the angular velocity in yaw, ψ and ψ_d are the yaw angle and (constant) desired yaw angle, respectively, $\delta \in \mathbb{R}$ is the rudder angle, $y \in \mathbb{R}$ is the roll angle measurement ($\boldsymbol{c} = [1, 0, 0, 0, 0]^T$), $w_H \in \mathbb{R}$ is used to describe 1st-order wave disturbances whereas $\boldsymbol{b}^*$ and $\boldsymbol{f}^*(\boldsymbol{x})$ depends on the ship.

The estimated model of the ship dynamics used for control design is usually linear and is written

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u \\ y &= \boldsymbol{c}^T\boldsymbol{x} \end{aligned} \tag{5}$$

where $u = \delta + w_H$. Assuming that $\boldsymbol{b}^* = \boldsymbol{b}$ is known and expanding the nonlinear term in (3), that is $\boldsymbol{f}^*(\boldsymbol{x}) = \boldsymbol{A}^*\boldsymbol{x} + \boldsymbol{g}^*(\boldsymbol{x})$, the ship dynamics can be rewritten as:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{\Delta}(\boldsymbol{x}) + \boldsymbol{b}u \\ y &= \boldsymbol{c}^T\boldsymbol{x} \end{aligned} \tag{6}$$

where $\boldsymbol{\Delta}(\boldsymbol{x}) = \boldsymbol{g}^*(\boldsymbol{x}) + (\boldsymbol{A}^* - \boldsymbol{A})\boldsymbol{x}$ is a unknown nonlinear term describing the modeling error.

Normal form

In order to analyze the nonlinear ship model the equations of motion are transformed to *normal form*, see Isidori (1989). Let r be the relative degree of the system (the number of time differentiations of the output y before the input u explicitly appears). It is easy to show that the ship model has relative degree 2. It is also convenient to introduce the notation $L_f\lambda(\boldsymbol{x}) = \frac{\partial\lambda}{\partial\boldsymbol{x}}\boldsymbol{f}(\boldsymbol{x})$ for the derivative of λ along f (Lie derivative). Hence the notation $L_f^k\lambda$ can be used to denote the recursion:

$$L_f^k\lambda(\boldsymbol{x}) = \frac{\partial L_f^{k-1}\lambda}{\partial\boldsymbol{x}}\boldsymbol{f}(\boldsymbol{x}) \tag{7}$$

The system (3) is transformed to normal form by applying the transformation $\boldsymbol{z} = \boldsymbol{\phi}(\boldsymbol{x}) = [\phi_1(\boldsymbol{x}), \cdots, \phi_r(\boldsymbol{x}), \phi_{r+1}(\boldsymbol{x}), \cdots, \phi_n(\boldsymbol{x})]^T$ where the first two transformations are defined as:

$$z_1 = \phi_1(\boldsymbol{x}) = \boldsymbol{c}^T\boldsymbol{x}, \quad z_2 = \phi_2(\boldsymbol{x}) = L_f\boldsymbol{c}^T\boldsymbol{x} \tag{8}$$

The inverse transformation will be denoted $\boldsymbol{x} = \boldsymbol{\phi}^{-1}(\boldsymbol{z})$. Applying this transformation to (3) yields the normal equations:

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= L_f^2 h(\boldsymbol{\phi}^{-1}(\boldsymbol{z})) + L_g L_f h(\boldsymbol{\phi}^{-1}(\boldsymbol{z}))u \\ \dot{z}_i &= L_f\phi_i(\boldsymbol{\phi}^{-1}(\boldsymbol{z})) + L_g\phi_i(\boldsymbol{\phi}^{-1}(\boldsymbol{z}))u \\ y &= z_1\end{aligned} \tag{9}$$

where $i = 3, 4, 5$. This system can be further simplified by choosing the last last three transformations such that

$$L_g\phi_i(\boldsymbol{\phi}^{-1}(\boldsymbol{z})) = 0, \quad i = 3, 4, 5 \tag{10}$$

This implies that the input u is eliminated from the last three differential equations. In order to simplify the notation system (9) is written as two subsystems with two and three state variables by defining $\boldsymbol{\xi} = [z_1, z_2]^T$ and $\boldsymbol{\eta} = [z_3, z_4, z_5]^T$. Hence if ϕ_i, $i = 3, 4, 5$, are chosen according to (10) then

$$\begin{bmatrix}\boldsymbol{\xi} \\ \boldsymbol{\eta}\end{bmatrix} = \begin{bmatrix}\phi \\ p \\ \cdots\cdots\cdots \\ (\psi - \psi_d) \\ b_2 p - b_1 v \\ b_3 p - b_1 r\end{bmatrix} \tag{11}$$

and (9) can be expressed as:

$$\begin{aligned}\dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= a^*(\boldsymbol{\xi},\boldsymbol{\eta}) + b^*(\boldsymbol{\xi},\boldsymbol{\eta})\delta \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{q}^*(\boldsymbol{\xi},\boldsymbol{\eta}) \\ y &= \xi_1\end{aligned} \tag{12}$$

where $a^*(\boldsymbol{\xi},\boldsymbol{\eta}) = L_f^2 h(\boldsymbol{\phi}^{-1}(\boldsymbol{z}))$ and $b^*(\boldsymbol{\xi},\boldsymbol{\eta}) = L_g L_f h(\boldsymbol{\phi}^{-1}(\boldsymbol{z}))$. Expanding (12) and using the fact that $L_g L_f h(\boldsymbol{\phi}^{-1}(\boldsymbol{z})) = b$, the following equations are obtained:

$$\begin{aligned}\dot{\xi}_1 &= \xi_2 \\ \dot{\xi}_2 &= \boldsymbol{r}^T\boldsymbol{\xi} + \boldsymbol{s}^T\boldsymbol{\eta} + a'(\boldsymbol{\xi},\boldsymbol{\eta}) + bu \\ \dot{\boldsymbol{\eta}} &= \boldsymbol{P}\boldsymbol{\xi} + \boldsymbol{Q}\boldsymbol{\eta} + \boldsymbol{q}'(\boldsymbol{\xi},\boldsymbol{\eta}) \\ y &= \xi_1\end{aligned} \tag{13}$$

where the linear terms corresponding to the linear term in (6) are known and the nonlinear terms corresponding to the nonlinear term of the dynamics in (6) are unknown.

Zero Dynamics

Normal form implies that the *zero-dynamics* of the system state-space model is extracted from the model. For the system (12) a zero output $y = 0$ corresponding to $\boldsymbol{\xi} = \boldsymbol{0}$ is obtained by choosing the control law as $u = -a(\boldsymbol{0},\boldsymbol{\eta})/b(\boldsymbol{0},\boldsymbol{\eta})$. Hence the $\boldsymbol{\eta}$-dynamics takes the form:

$$\dot{\boldsymbol{\eta}} = \boldsymbol{q}^*(\boldsymbol{0},\boldsymbol{\eta}) \tag{14}$$

which simply is the system zero-dynamics. If the zero-dynamics is unstable the system (3) is said to be *nonlinear non-minimum phase*. From (13) or (14) it is easily verified that the zero-dynamics is independent of the input, and cannot be altered by feedback. Thus if the zero-dynamics is unstable, perfect tracking is impossible and the goal becomes asymptotic tracking.

Assumptions

It is well known that all ships show a non-minimum phase behaviour in roll whereas yaw is minimum phase, see Fossen and Lauvdal (1994). This implies that the zero-dynamics given by the input u and and output $y = \boldsymbol{c}^T\boldsymbol{x}$ is unstable. Hence, output feedback linearization cannot be used in roll since the inverse dynamics will be unstable. The proposed controller in this paper is a nonlinear roll damping control law which can be used for non-minimum phase systems and it is based on Assumptions 1 and 2.

Assumption 1. *The zero-dynamics is unstable if and only if*

$$\boldsymbol{Q} = \frac{\partial}{\partial\boldsymbol{\eta}}\boldsymbol{q}^*(\boldsymbol{0},\boldsymbol{\eta})\Big|_{\boldsymbol{\eta}=\boldsymbol{0}} \tag{15}$$

has eigenvalues with positive real part.

This simply states that unstability is caused by the linear part of the dynamics.

Now if $\dot{\boldsymbol{\eta}} = \boldsymbol{q}^*(\boldsymbol{0},\boldsymbol{\eta})$ is the zero-dynamics of the ship dynamics and $\dot{\boldsymbol{\eta}} = \boldsymbol{Q}\boldsymbol{\eta}$ is the estimated zero-dynamics, a 2nd assumption is introduced:

Assumption 2. *$\dot{\boldsymbol{\eta}} = \boldsymbol{q}^*(\boldsymbol{0},\boldsymbol{\eta})$ is unstable if and only if $\dot{\boldsymbol{\eta}} = \boldsymbol{Q}\boldsymbol{\eta}$ is unstable.*

This is a rather weak assumtion since the linear estimate of the dynamics usually is close to the actual linear dynamics and, by Assumtion 1, the unstability is caused by the linear part exclusively.

3. DESIGN OF A NON-MINIMUM PHASE RUDDER-ROLL STABILIZATOR AND COURSE-KEEPING CONTROLLER

3.1. Sliding Surface in Roll

Define a sliding surface, see Gopalswamy and Hedrick (1993):

$$\begin{aligned} \sigma &= \xi_2 + \lambda\xi_1 & (16)\\ &= \dot{y} + \lambda y & (17)\end{aligned}$$

where ξ_2, ξ_1 and y are defined in (13) and $\lambda > 0$. Taking the sliding surface as the output, the ship dynamics can be rewritten as:

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{\Delta}(\boldsymbol{x}) + \boldsymbol{b}u \\ \sigma &= x_1 + \lambda x_2 \end{aligned} \quad (18)$$

The control objective is to regulate σ to zero such that $\phi = p = 0$. It is easily verified that the modified ship model (18) has relative degree 1. Thus when defining a new vector

$$\tilde{\boldsymbol{\eta}} = [\xi_1 \ , \ \boldsymbol{\eta}^T]^T \quad (19)$$

it is seen that (13) with

$$\boldsymbol{P}\boldsymbol{\xi} = [\boldsymbol{p}_{\xi_1}, \boldsymbol{p}_{\xi_2}] \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}, \boldsymbol{r}^T\boldsymbol{\xi} = [r_{\xi_1}, r_{\xi_2}] \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \quad (20)$$

can be written in normal form:

$$\begin{aligned} \dot{\sigma} &= \ddot{y} + \lambda\dot{y} \\ &= \dot{\xi}_2 + \lambda\dot{\xi}_1 \\ &= \lambda\xi_2 + \boldsymbol{r}^T\boldsymbol{\xi} + \boldsymbol{s}^T\boldsymbol{\eta} + a'(\boldsymbol{\xi},\boldsymbol{\eta}) + bu & (21)\\ &\Downarrow \\ \dot{\sigma} &= \tilde{r}\sigma + \tilde{\boldsymbol{s}}^T\tilde{\eta} + \tilde{a}'(\sigma,\tilde{\boldsymbol{\eta}}) + bu & (22)\\ \dot{\tilde{\boldsymbol{\eta}}} &= \tilde{\boldsymbol{p}}\sigma + \tilde{\boldsymbol{Q}}\tilde{\eta} + \tilde{\boldsymbol{q}}'(\sigma,\tilde{\boldsymbol{\eta}}) & (23)\end{aligned}$$

where

$$\begin{aligned} \tilde{r} &= (\lambda + r_{\xi_2}) \\ \tilde{\boldsymbol{s}} &= [(r_{\xi_1} - \lambda r_{\xi_2} - \lambda^2) \ , \ \boldsymbol{s}^T]^T \\ \tilde{\boldsymbol{p}} &= [1 \ , \ \boldsymbol{p}_{\xi_2}^T]^T \\ \tilde{\boldsymbol{Q}} &= \begin{bmatrix} -\lambda & \boldsymbol{0} \\ (\boldsymbol{p}_{\xi_1} - \lambda\boldsymbol{p}_{\xi_2}) & \boldsymbol{Q} \end{bmatrix} \end{aligned} \quad (24)$$

It is easily verified that the linear part of the zero-dynamics, $\dot{\tilde{\boldsymbol{\eta}}} = \tilde{\boldsymbol{Q}}\tilde{\boldsymbol{\eta}}$ still is unstable. This problem can, however, be solved by modifying the sliding surface.

3.2. Combined Sliding Surface for Roll and Yaw

Consider the modified sliding surface, see Gopalswamy and Hedrick (1993):

$$\tilde{\sigma} = \sigma - \boldsymbol{k}^T\tilde{\boldsymbol{\eta}} \quad (25)$$

where $\boldsymbol{k} = [\,k_1 \ , \ k_2 \ , \ k_3 \ , \ k_4]^T$. Hence the modified $\tilde{\sigma}$- and $\tilde{\boldsymbol{\eta}}$-dynamics are:

$$\begin{aligned} \dot{\tilde{\sigma}} &= (\tilde{r} - \boldsymbol{k}^T\tilde{\boldsymbol{p}})\tilde{\sigma} + \tilde{a}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}},\tilde{\boldsymbol{\eta}}) \\ &\quad + (\tilde{r}\boldsymbol{k}^T + \tilde{\boldsymbol{s}}^T - \boldsymbol{k}^T(\tilde{\boldsymbol{p}}\boldsymbol{k}^T + \tilde{\boldsymbol{Q}}))\tilde{\eta} \\ &\quad - \boldsymbol{k}^T\tilde{\boldsymbol{q}}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}},\tilde{\boldsymbol{\eta}}) + bu & (26)\\ \dot{\tilde{\boldsymbol{\eta}}} &= \tilde{\boldsymbol{p}}\tilde{\sigma} + (\tilde{\boldsymbol{p}}\boldsymbol{k}^T + \tilde{\boldsymbol{Q}})\tilde{\boldsymbol{\eta}} + \tilde{\boldsymbol{q}}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}},\tilde{\boldsymbol{\eta}}) & (27)\end{aligned}$$

From (14), (26) and (27) the zero dynamics in $(\tilde{\sigma},\tilde{\boldsymbol{\eta}})$-coordinates are obtained:

$$\dot{\tilde{\boldsymbol{\eta}}} = (\tilde{\boldsymbol{p}}\boldsymbol{k}^T + \tilde{\boldsymbol{Q}})\tilde{\boldsymbol{\eta}} + \tilde{\boldsymbol{q}}'(\boldsymbol{k}^T\tilde{\boldsymbol{\eta}},\tilde{\boldsymbol{\eta}}) \quad (28)$$

Under Assumptions 1 and 2 it is concluded that (28) is stable if $(\tilde{\boldsymbol{p}},\tilde{\boldsymbol{Q}})$ is a controllable pair (it can be shown that this is the case if $(\boldsymbol{A},\boldsymbol{b})$ in (6) is a controllable pair) and $\boldsymbol{k}^T$ stabilizes the linear part of the zero-dynamics, i.e.:

$$\mathrm{Re}\left\{\det\left(\lambda\boldsymbol{I} - (\tilde{\boldsymbol{p}}\boldsymbol{k}^T + \tilde{\boldsymbol{Q}})\right)\right\} < 0 \quad (29)$$

Hence regulation of $\tilde{\sigma}$ to zero implies that $\tilde{\boldsymbol{\eta}} \to \boldsymbol{0}$ and finally (25) implies that $\sigma \to 0$, i.e.

$$\tilde{\sigma} \to 0 \ \Rightarrow \ \phi, \ p, \ v, \ r, \ (\psi - \psi_d) \to 0 \quad (30)$$

3.3. Nonlinear Control Law

In order to obtain the desired $\tilde{\sigma}$-dynamics ($\tilde{\sigma} \to 0$ as $t \to \infty$), we propose the following controller:

$$\delta = \frac{1}{b}\left(k_{\tilde{\sigma}}\tilde{\sigma} + \boldsymbol{k}_{\tilde{\eta}}^T\tilde{\boldsymbol{\eta}} - \mu\tanh(\varphi\tilde{\sigma})\right) \quad (31)$$

Here $k_{\tilde{\sigma}}$ is the feedback gain defined as

$$k_{\tilde{\sigma}} = \boldsymbol{k}^T\tilde{\boldsymbol{p}} - \tilde{r} - \gamma \quad (32)$$

where $\gamma \geq 0$, $\boldsymbol{k}_{\tilde{\eta}}$ is a feedforward gain vector,

$$\boldsymbol{k}_{\tilde{\eta}}^T = \boldsymbol{k}^T(\tilde{\boldsymbol{p}}\boldsymbol{k}^T - \tilde{\boldsymbol{Q}}) - \tilde{r}\boldsymbol{k}^T - \tilde{\boldsymbol{s}}^T \quad (33)$$

whereas $\mu > 0$ is the switching gain and $\varphi > 0$ is a boundary layer parameter. The definition of the switching gain is given by the stability proof in Section 3.4 while the boundary layer parameter is a design parameter limiting the maximum value of the time derivative of the commanded rudder.

The ship with control law is shown in Fig. 1.

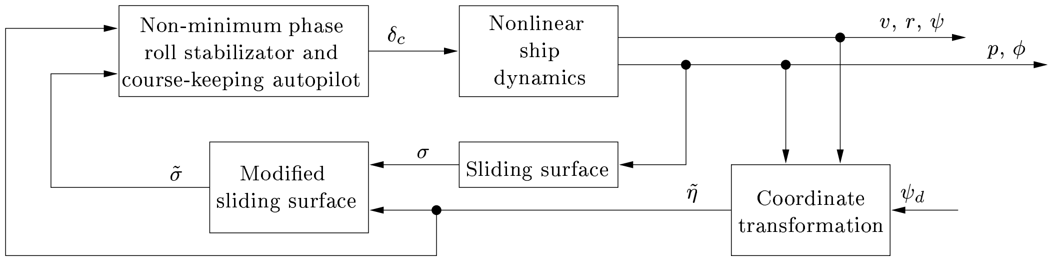

Fig. 1: Block diagram showing the roll damping system and course-keeping controller.

3.4. Stability Proof

Since the zero-dynamics (28) is stable and independant of the control variable, δ, then according to Isidori (1989) $\tilde{\boldsymbol{\eta}} \to \mathbf{0}$ if $\tilde{\sigma} = 0$. Hence stablilty of (26) and (27), and thus the ship model (3), with the controller (31) can be proven by using the following Lyapunov-function candidate:

$$V\left(\tilde{\sigma}(\boldsymbol{x})\right) = \frac{1}{2}\tilde{\sigma}^2(\boldsymbol{x}) \geq 0 \tag{34}$$

Writing $\tilde{\sigma}(\boldsymbol{x}) = \tilde{\sigma}$ the time derivative of the Lyapunov-function canditate becomes:

$$\dot{V}(\tilde{\sigma}) = \tilde{\sigma}\dot{\tilde{\sigma}} \tag{35}$$

Substitution of the $\tilde{\sigma}$-dynamics (26) into $\dot{V}$ implies:

$$\begin{aligned}\dot{V}(\tilde{\sigma}) &= \tilde{\sigma}(\tilde{r} - \boldsymbol{k}^T\tilde{\boldsymbol{p}})\tilde{\sigma} + \tilde{\sigma}\tilde{a}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}}) \\ &\quad + \tilde{\sigma}\left(\tilde{r}\boldsymbol{k}^T + \tilde{\boldsymbol{s}}^T - \boldsymbol{k}^T(\tilde{\boldsymbol{p}}\boldsymbol{k}^T + \tilde{\boldsymbol{Q}})\right)\tilde{\eta} \\ &\quad - \tilde{\sigma}\boldsymbol{k}^T\tilde{\boldsymbol{q}}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}}) + \tilde{\sigma}b\delta\end{aligned} \tag{36}$$

Substituting the controller[1] (31) into this expression yields:

$$\begin{aligned}\dot{V}(\tilde{\sigma}) &= \tilde{\sigma}(\tilde{r} - \boldsymbol{k}^T\tilde{\boldsymbol{p}} + k_{\tilde{\sigma}})\tilde{\sigma} - \tilde{\sigma}\mu \cdot \mathrm{sign}(\tilde{\sigma}) \\ &\quad + \tilde{\sigma}\left(\tilde{r}\boldsymbol{k}^T + \tilde{\boldsymbol{s}}^T - \boldsymbol{k}^T(\tilde{\boldsymbol{p}}\boldsymbol{k}^T + \tilde{\boldsymbol{Q}}) + \boldsymbol{k}_{\tilde{\eta}}^T\right)\tilde{\eta} \\ &\quad + \tilde{\sigma}\tilde{a}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}}) + \tilde{\sigma}bw_H \\ &\quad - \tilde{\sigma}\boldsymbol{k}^T\tilde{\boldsymbol{q}}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}})\end{aligned} \tag{37}$$

Now, since $\boldsymbol{A}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are known (and thus $\tilde{\boldsymbol{Q}}$, $\tilde{\boldsymbol{s}}$, $\tilde{\boldsymbol{p}}$ and $\tilde{r}$), the choise of control parameters given by (32) and (33) implies that

$$\begin{aligned}\dot{V}(\tilde{\sigma}) &= -\gamma\tilde{\sigma}^2 - \tilde{\sigma}\mu \cdot \mathrm{sign}(\tilde{\sigma}) \\ &\quad + \tilde{\sigma}\tilde{a}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}}) + \tilde{\sigma}bw_H \\ &\quad - \tilde{\sigma}\boldsymbol{k}^T\tilde{\boldsymbol{q}}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}})\end{aligned} \tag{38}$$

Finally, by chosing the switching gain μ according to

$$\begin{aligned}\mu &> \|\boldsymbol{k}^T\tilde{\boldsymbol{q}}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}})\|_2 + \|bw_H\|_2 \\ &\quad + \|\tilde{a}'(\tilde{\sigma} + \boldsymbol{k}^T\tilde{\boldsymbol{\eta}}, \tilde{\boldsymbol{\eta}})\|_2\end{aligned} \tag{39}$$

a negative semi-definite time derivative of the Lyapunov-function canditate $V(\tilde{\sigma})$ is obtained, that is:

$$\dot{V}(\tilde{\sigma}) \leq 0 \tag{40}$$

Since $V(\tilde{\sigma}) \geq 0$ and $\dot{V}(\tilde{\sigma}) \leq 0$ all states are bounded, and application of LaSalle's invariant set theorem implies that (Khalil (1992))

$$\tilde{\sigma} \to 0 \quad \text{as} \quad t \to \infty \tag{41}$$

4. CASE STUDY

The controller is applied to the nonlinear ship model given in Blanke and Christensen (1993). Some of the main ship data are given in Appendix A. The rudder saturation δ_{max} is set to 30 (deg) while the rudder rate $\dot{\delta}_{max}$ is limited to 15 (deg/s). A measure of effectiveness of the rudder-roll damping system is taken from Oda et al. (1992) and reads as follows:

$$\text{Roll reduction} = 100 \cdot \frac{\text{AP} - \text{RRCS}}{\text{AP}}\% \tag{42}$$

where RRCS and AP are the standard deviations with and without roll damping, respectively. The vector $\boldsymbol{k}$ in the transformation:

$$\tilde{\sigma} = \sigma - \boldsymbol{k}^T\tilde{\boldsymbol{\eta}} \tag{43}$$

was computed using the **matlab** pole-placement function *acker* and the eigenvalues of

$$\lambda_i\{\tilde{\boldsymbol{p}}\boldsymbol{k}^T + \tilde{\boldsymbol{Q}}\} \tag{44}$$

were placed at -0.5, i.e. $\lambda_i = -0.5, \quad i = 1,2,3,4$. The feedback and feedforward gains, $k_{\tilde{\sigma}}$ and $\boldsymbol{k}_{\tilde{\eta}}$, in (31) were chosen according to (32) and (33). Finally, $\gamma = 0$ whereas the switching gain μ and the boundary layer parameter φ was set to 0.2 and 1, respectively.

In the first simulation the cruise speed was set to 10 m/s and the states of interest are shown in Figure 2. By comparing the roll angle ϕ and the angular velocity in roll p with a simulation without rudder-roll damping there were obtained

[1] *Notice that* $\tanh(\varphi\tilde{\sigma})$ *is replaced by* $\mathrm{sign}(\tilde{\sigma})$ *in the stability proof in order to guarantee that* $\dot{V} \leq 0$. *The continious function* $\tanh(\varphi\tilde{\rho})$ *is, however, used for implementation to reduce chattering and does not cause stability problems.*

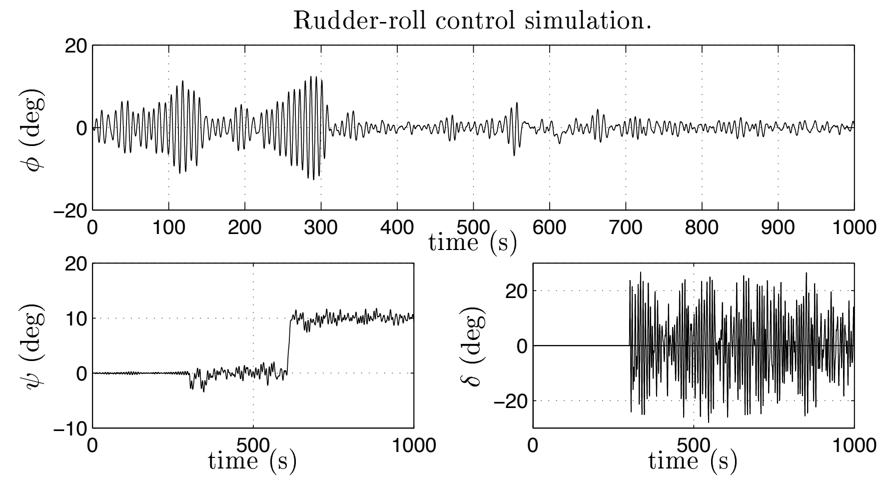

Fig. 2: The figure shows the roll angle ϕ, yaw angle ψ and rudder angle δ in a simulation over 1000 (s) when the cruise speed U is 10 (m/s). The controller is not active the first 300 (s) and is turned on after 300 (s). The reference angle in yaw changes from 0 (deg) to 10 (deg) after 600 (s). The wave encounter frequency is 0.8 (rad/s).

a damping in roll and roll rate of 71 % and 73%, respectively. These are remarkable good results.

Figure 3 shows the same controller (no speed-scaling) when the cruise speed is reduced to 8 (m/s). Naturally the effectiveness is reduced and the damping where measured to be 50 % in roll and 49 % in roll rate. This can be improved by scaling the controller with respect to speed. Notice that the autopilot in yaw has exellent performance even without speed scaling.

5. CONCLUSIONS

The combined rudder-roll damping and course controller derived in this paper proved to be easy to tune and the stability in the presence of 1st-order wave disturbances has been verified by simulations. The robustness to modeling errors has been demonstrated by applying the same controller at two different cruise speeds.

6. ACKNOWLEDGEMENT

This work was sponsored by the Center of Maritime Research at the Department of Engineering Cybernetics, Norwegian University of Science and Technology and SINTEF Automatic Control.

A. SHIP MODEL DATA

A detailed description of the nonlinear ship model used in the simulations is found in Blanke and Christensen (1993). Some of the main ship data are given in Tab. 1.

Main particulars for ship		
Length, L_{pp}	48	m
Beam, B	8.6	m
Draft, D	2.2	m
Displacement, ∇	350	m^3
Inertia in yaw, I_z	$33.7 \cdot 10^6$	$kg\,m^2$
Inertia in roll, I_x	$3.4 \cdot 10^6$	$kg\,m^2$

Tab. 1: Data for the nonlinear ship model.

B. REFERENCES

Baitis, E., D. A. Woolaver and T. A. Beck (1983). Rudder Roll Stabilization for Coast Guards Cutters and Frigates. *Naval Engineers Journal,* pp. 267–282.

Baitis, E., D. A. Woolaver and T. A. Beck (1989). Ship Roll Stabilization in the U. S. Navy. *Naval Engineers Journal,* pp. 43–53.

Blanke, M. and A. Christensen (1993). Rudder-Roll Damping Autopilot Robustness due to Sway-Yaw-Roll Couplings. In: *Proceedings of the 10th International Ship Control Systems Symposium (SCSS'93).* Ottawa, Canada. pp. A.93–A.119.

Blanke, M., P. Haals and K. K. Andreasen (1989). Rudder Roll Damping Experience in Denmark. In: *IFAC Workshop on Expert Systems and Signal Processing in Marine Automation.* Lyngby, Denmark. pp. 149–160.

Fossen, T. I. (1994). *Guidance and Control of Ocean Vehicles.* John Wiley and Sons Ltd.

Fossen, T. I. and T. Lauvdal (1994). Nonlinear Stability Analysis of Ship Autopilots in Sway,

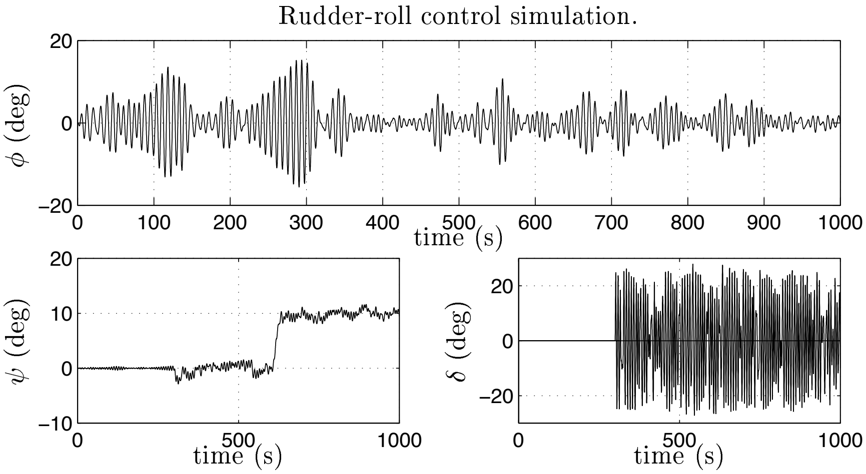

Fig. 3: The figure shows the roll angle ϕ, yaw angle ψ and rudder angle δ in a simulation over 1000 (s) when the cruise speed U is 8 (m/s). The controller is not active the first 300 (s) and is turned on after 300 (s). The reference angle in yaw changes from 0 (deg) to 10 (deg) after 600 (s). The wave encounter frequency is 0.8 (rad/s).

Roll and Yaw. In: *Proceedings of the 3rd Conference on Marine Craft Maneuvering and Control (MCMC'94), Southampton, UK*. pp. 113–124.

Gopalswamy, S. and J. K. Hedrick (1993). Tracking Nonlinear Non-minimum Phase Systems using Sliding Control. *Int. J. Control* **57**(5), 1141–1158.

Healey, A. J. and D. Lienard (1993). Multivariable Sliding Mode Control for Autonomous Diving and Steering of Unmanned Underwater Vehicles. *IEEE Journal of Ocean Engineering* **OE-18**(3), 327–339.

Isidori, A. (1989). *Nonlinear Control Systems*. Springer-Verlag. Berlin.

Källström, C. G. (1987). Improved Operational Effectiveness of Naval Ships by Rudder Roll Stabilization. In: *NAVAL'87, Asian Pacific Naval Exhibition and Conference*. Singapore.

Källström, C. G. and W. L. Schultz (1990). An Integrated Rudder Control System for Roll Damping and Maintenance. In: *Proceedings of the 9th International Ship Control Systems Symposium (SCSS'90)*. Bethesda, MD. pp. 3.278–3.296.

Källström, C. G., P. Wessel and S. Sjölander (1988). Roll Reduction by Rudder Control. In: *SNAME Spring Meeting/STAR Symposium*. Pittsburg, Pennsylvania. pp. 67–76.

Katebi, M. R., D. K. K. Wong and M. J. Grimble (1987). LQG Autopilot and Rudder Roll Stabilization Control System Design. In: *Proceedings of the 8th International Ship Control Systems Symposium (SCSS'87)*. The Hague, The Netherlands. pp. 3.69–3.84.

Khalil, K. K. (1992). *Nonlinear Systems*. Macmillian Publishing Company.

Oda et al., H. (1992). Rudder Roll Stabilization Control System through Multivariable Auto Regressive Model. In: *Proceedings of the 3rd IFAC Workshop on Control Applications in Marine Systems (CAMS'92)*. Genova, Italy. pp. 113–127.

Van Amerongen, J. and H. R. Van Nauta Lempke (1987). Adaptive Control Aspects of a Rudder Roll Stabilization System. In: *Proceedings of the 10th IFAC World Congress*. Munich, Germany. pp. 215–219.

Van der Klugt, P. G. M. (1987). Rudder Roll Stabilization. PhD thesis. Delft University of Technology, The Netherlands.

Zhou, W. W. (1990). A New Approach for Adaptive Rudder Roll Stabilization Control. In: *Proceedings of the 9th International Ship Control Systems Symposium (SCSS'90)*. Bethesda, MD. pp. 1.115–1.125.