Nonlinear Output Feedback Control of Dynamically Positioned Ships Using Vectorial Observer Backstepping

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Abstract— **Dynamic positioning (DP) systems for ships are usually designed under the assumption that the kinematic equations can be linearized about a constant yaw angle such that linear theory and gain scheduling techniques can be applied. This paper proposes a globally exponentially stable (GES) nonlinear control law where this assumption is removed. A nonlinear observer is included in the design such that only position measurements are required. GES is proven by applying the backstepping design methodology and Lyapunov stability theory. The control law is simulated on two thruster controlled ships.**

I. Introduction

A positioned ship is controlled in three degrees of freedom (surge, sway and yaw) by means of main propellers aft of the ship, tunnel thrusters and azimuth (rotatable) thrusters mounted under the hull. Rudders are not used during station-keeping since the rudder forces are quite small at low speed. Conventional DP-systems are designed by linearizing the kinematic equations of motions about a set of predefined constant yaw angles such that linear optimal control theory and gain-scheduling techniques can be applied. The kinematic equations of motion are typically linearized about 36 different yaw angles (steps of 10 degrees to cover 360 degrees). For each of these linearized models, optimal Kalman filter and feedback control gains are computed. The Kalman filter is necessary to produce noise-free estimates of the velocities and positions since only positions are measured, see [1], [5], [9] and [10].

The main motivation for this paper is to remove these assumptions by using nonlinear observer and feedback control theory. Linearization of the kinematics is avoided by using a modified version of the nonlinear observer presented in [3]. Next, nonlinear feedback from the state estimates is obtained by using the observer backstepping design methodology [7]. The results of [7] are further improved by replacing the measured output with a filtered output when designing the feedback control law. Hence, the control inputs are generated by using filtered estimates of both the velocities and positions. Finally, GES is proven for the total system, that is ship model, observer and control system.

The control law presented in this paper was initially derived on component form [6] by using 6 steps (3 for both the position and velocity components) when performing the backstepping. In this paper it is shown that the number of steps can be reduced to 2 by approaching the problem in a vector setting. It should be noted that the proposed nonlinear control law is only a first attempt towards the design of a fully nonlinear DP-control system since extensions to wave filtering and bias state estimation also must be made.

A. Ship Model

In the forthcoming sections, the earth-fixed positions (x, y) and yaw angle ψ of the vessel is expressed in vector form according to $\boldsymbol{\eta} = [x, y, \psi]^T$ whereas the body-fixed velocities are represented by the vector $\boldsymbol{\nu} = [u, v, r]^T$. The elements in these state vectors describes the surge, sway and yaw modes, respectively.

B. Equations of Motion

The derivation of the ship model is based on [2]. The case study of this paper is a dynamically positioned ship which is controlled exclusively by means of thrusters. The ship can also be supplied with anchors if thruster assisted mooring is to be considered. The mooring forces are modeled as spring forces, $\mathbf{K}(\boldsymbol{\eta} - \boldsymbol{\eta}_0)$ where $\boldsymbol{\eta}_0$ is the equilibrium position of the mooring system. For notational simplicity it is assumed that $\boldsymbol{\eta}_0 = \mathbf{0}$.

During DP the damping forces can be assumed to be linear since the speed of the ship is quite small. Hence the equations of motion in surge, sway and yaw can be written:

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{D}\boldsymbol{\nu} + \mathbf{K}\boldsymbol{\eta} = \boldsymbol{\tau} \tag{1}$$

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{2}$$

where

$$\mathbf{J}(\boldsymbol{\eta}) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

is the rotation matrix in yaw and $\mathbf{M}$ is the inertia matrix including hydrodynamic added inertia. The control forces and moment $\boldsymbol{\tau} = [\tau_1, \tau_2, \tau_3]^T$ are provided by the thruster system, see [2] for details. The matrices $\mathbf{M}, \mathbf{D}$ and $\mathbf{K}$ for a offshore supply vessel and a floating production ship are given in Section V. Starboard-port symmetry of ships implies that $\mathbf{M}$ and $\mathbf{D}$ takes the following structure:

$$\mathbf{M} = \begin{bmatrix} m_{11} & 0 & 0 \\ 0 & m_{22} & m_{23} \\ 0 & m_{32} & m_{33} \end{bmatrix} \mathbf{D} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & d_{32} & d_{33} \end{bmatrix} \tag{4}$$

that is, there is no coupling between the surge and the sway-yaw subsystems. The anchor forces and moment are usually described by a diagonal matrix:

$$\mathbf{K} = \operatorname{diag}\{k_{11}, k_{22}, k_{33}\}. \tag{5}$$

In general $\mathbf{M}$ will be non-symmetrical, that is $m_{23} \neq m_{32}$ due to the properties of hydrodynamic added inertia [2]. In fact, hydrodynamic added mass will depend on the speed of the ship and the wave frequency of encounter. However, for low-speed and zero speed applications e.g. dynamic positioning, the inertia matrix $\mathbf{M} = \mathbf{M}^T$ is *positive definite* and *constant*. The damping matrix $\mathbf{D}$ is, however, non-symmetrical in most cases. This will not affect the dissipative nature of ships since the quadratic term $\mathbf{x}^T\mathbf{D}\mathbf{x} = \frac{1}{2}\mathbf{x}^T(\mathbf{D} + \mathbf{D}^T)\mathbf{x} > 0$ for all $\mathbf{x} \neq \mathbf{0}$, that is $\mathbf{D}$ is *strictly positive.*

C. Resulting State-Space Model

The resulting system model is written as:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{6}$$

$$\dot{\boldsymbol{\nu}} = \mathbf{A}_1\boldsymbol{\eta} + \mathbf{A}_2\boldsymbol{\nu} + \mathbf{B}\boldsymbol{\tau} \tag{7}$$

where

$$\mathbf{A}_1 = -\mathbf{M}^{-1}\mathbf{K},\ \mathbf{A}_2 = -\mathbf{M}^{-1}\mathbf{D},\ \mathbf{B} = \mathbf{M}^{-1} \tag{8}$$

It is assumed that only position and yaw angle measurements are available, that is

$$\mathbf{y} = \boldsymbol{\eta} \tag{9}$$

II. Nonlinear Observer Design

The nonlinear observer is found by using Lyapunov theory which puts constraints on the choice of the filter gains. This is based on [3], where a nonlinear model-based observer for an underwater vehicle with filter gains which are functions of the measured attitude is proposed. The yaw angle ψ is assumed to be measured with good accuracy by using a gyro compass. The position measurements x and y are assumed measured by using e.g. the NAVSTAR differential global positioning system (DGPS) which is a world wide satellite navigation systems. The DGPS-receiver is usually configured to produce position measurements at 1–10 (Hz) for marine applications.

An observer for (6) and (7) is constructed as:

$$\begin{aligned} \dot{\hat{\boldsymbol{\eta}}} &= \mathbf{J}(\mathbf{y})\hat{\boldsymbol{\nu}} + \mathbf{K}_1\tilde{\boldsymbol{\eta}} \quad (10) \\ \dot{\hat{\boldsymbol{\nu}}} &= \mathbf{A}_1\hat{\boldsymbol{\eta}} + \mathbf{A}_2\hat{\boldsymbol{\nu}} + \mathbf{B}\boldsymbol{\tau} + \mathbf{K}_2\tilde{\boldsymbol{\eta}} \quad (11) \end{aligned}$$

where $\tilde{\boldsymbol{\eta}} = \boldsymbol{\eta} - \hat{\boldsymbol{\eta}}$ is the position estimation error. Defining $\tilde{\boldsymbol{\nu}} = \boldsymbol{\nu} - \hat{\boldsymbol{\nu}}$, the error dynamics can be written:

$$\begin{aligned} \dot{\tilde{\boldsymbol{\eta}}} &= \mathbf{J}(\mathbf{y})\tilde{\boldsymbol{\nu}} - \mathbf{K}_1\tilde{\boldsymbol{\eta}} \quad (12) \\ \dot{\tilde{\boldsymbol{\nu}}} &= (\mathbf{A}_1 - \mathbf{K}_2)\tilde{\boldsymbol{\eta}} + \mathbf{A}_2\tilde{\boldsymbol{\nu}} \quad (13) \end{aligned}$$

Notice that the measured yaw angle ψ is used to compute $\mathbf{J}(\mathbf{y})$. The computation of $\mathbf{J}(\mathbf{y})$ is quite accurate since the gyro compass measurement noise will be less than 0.1 (deg). However, good filtering of x and y is important since DGPS-measurement noise will be in the range of 1–3 (m). Therefore, the feedback control law will be based on the filtered signals $\hat{\boldsymbol{\eta}}$ and $\hat{\boldsymbol{\nu}}$ which both have to be produced from the noisy signal $\boldsymbol{\eta}$.

The matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ in (10) and (11) can be chosen such that the observer is GES. This is obtained by defining a Lyapunov function candidate:

$$V_{obs} = \frac{1}{2}\left(\tilde{\boldsymbol{\eta}}^T\mathbf{P}_1\tilde{\boldsymbol{\eta}} + \tilde{\boldsymbol{\nu}}^T\mathbf{P}_2\tilde{\boldsymbol{\nu}}\right) > 0, \ \forall\tilde{\boldsymbol{\eta}} \neq \mathbf{0}, \tilde{\boldsymbol{\nu}} \neq \mathbf{0} \quad (14)$$

where $\mathbf{P}_1 = \mathbf{P}_1^T$ and $\mathbf{P}_2 = \mathbf{P}_2^T$ are positive definite matrices. Hence:

$$\begin{aligned} \dot{V}_{obs} &= \dot{\tilde{\boldsymbol{\eta}}}^T\mathbf{P}_1\tilde{\boldsymbol{\eta}} + \frac{1}{2}\left(\dot{\tilde{\boldsymbol{\nu}}}^T\mathbf{P}_2\tilde{\boldsymbol{\nu}} + \tilde{\boldsymbol{\nu}}^T\mathbf{P}_2\dot{\tilde{\boldsymbol{\nu}}}\right) \\ &= \left(\mathbf{J}(\mathbf{y})\tilde{\boldsymbol{\nu}} - \mathbf{K}_1\tilde{\boldsymbol{\eta}}\right)^T\mathbf{P}_1\tilde{\boldsymbol{\eta}} \\ &\quad + \frac{1}{2}\tilde{\boldsymbol{\nu}}^T\left(\mathbf{P}_2\mathbf{A}_2 + \mathbf{A}_2^T\mathbf{P}_2\right)\tilde{\boldsymbol{\nu}} \\ &\quad + \frac{1}{2}\tilde{\boldsymbol{\eta}}^T\left(\mathbf{A}_1 - \mathbf{K}_2\right)^T\mathbf{P}_2\tilde{\boldsymbol{\nu}} \quad (15) \\ &\quad + \frac{1}{2}\tilde{\boldsymbol{\nu}}^T\mathbf{P}_2\left(\mathbf{A}_1 - \mathbf{K}_2\right)\tilde{\boldsymbol{\eta}} \\ &= -\tilde{\boldsymbol{\eta}}^T\mathbf{K}_1^T\mathbf{P}_1\tilde{\boldsymbol{\eta}} + \frac{1}{2}\tilde{\boldsymbol{\nu}}^T\left(\mathbf{P}_2\mathbf{A}_2 + \mathbf{A}_2^T\mathbf{P}_2\right)\tilde{\boldsymbol{\nu}} \\ &\quad + \tilde{\boldsymbol{\nu}}^T\left(\mathbf{J}^T(\mathbf{y})\mathbf{P}_1 + \mathbf{P}_2\left(\mathbf{A}_1 - \mathbf{K}_2\right)\right)\tilde{\boldsymbol{\eta}} \end{aligned}$$

V_{obs} can be made negative definite by defining:

$$\mathbf{J}^T(\mathbf{y})\mathbf{P}_1 + \mathbf{P}_2\left(\mathbf{A}_1 - \mathbf{K}_2\right) \triangleq \mathbf{0} \quad (16)$$

$$\mathbf{K}_1^T\mathbf{P}_1 \triangleq \mathbf{Q}_1 \quad (17)$$

$$\frac{1}{2}\left(\mathbf{P}_2\mathbf{A}_2 + \mathbf{A}_2^T\mathbf{P}_2\right) \triangleq -\mathbf{Q}_2 \quad (18)$$

where $\mathbf{Q}_1 = \mathbf{Q}_1^T$ and $\mathbf{Q}_2 = \mathbf{Q}_2^T$ are positive definite design matrices and $\mathbf{A}_2 = -\mathbf{M}^{-1}\mathbf{D}$ is assumed to be *Hurwitz*. This implies that the ship must be course-stable. An extension to course-unstable ships can, however, be made by applying the approach of [8]. Hence:

$$\dot{V}_{obs} = -\tilde{\boldsymbol{\eta}}^T\mathbf{Q}_1\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{\nu}}^T\mathbf{Q}_2\tilde{\boldsymbol{\nu}} < 0, \ \forall\tilde{\boldsymbol{\eta}} \neq \mathbf{0}, \tilde{\boldsymbol{\nu}} \neq \mathbf{0} \quad (19)$$

GES of the observer can be proven by defining $\mathbf{x} = [\tilde{\boldsymbol{\eta}}^T, \tilde{\boldsymbol{\nu}}^T]^T$ and:

$$\begin{aligned} \mathbf{P} &= \mathrm{diag}\{\mathbf{P}_1, \mathbf{P}_2\} \quad (20) \\ \mathbf{Q} &= \mathrm{diag}\{\mathbf{Q}_1, \mathbf{Q}_2\} \quad (21) \end{aligned}$$

and noticing that:

$$\begin{aligned} V_{obs} &= \frac{1}{2}\left(\tilde{\boldsymbol{\eta}}^T\mathbf{P}_1\tilde{\boldsymbol{\eta}} + \tilde{\boldsymbol{\nu}}^T\mathbf{P}_2\tilde{\boldsymbol{\nu}}\right) \quad (22) \\ &= \frac{1}{2}\mathbf{x}^T\mathbf{P}\mathbf{x} \leq \frac{\lambda_{\max}(\mathbf{P})}{2}\mathbf{x}^T\mathbf{x} \end{aligned}$$

$$\begin{aligned} \dot{V}_{obs} &= -\tilde{\boldsymbol{\eta}}^T\mathbf{Q}_1\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{\nu}}^T\mathbf{Q}_2\tilde{\boldsymbol{\nu}} \quad (23) \\ &= -\mathbf{x}^T\mathbf{Q}\mathbf{x} \leq -\lambda_{\min}(\mathbf{Q})\mathbf{x}^T\mathbf{x} \end{aligned}$$

where $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalue, respectively. Hence:

$$V_{obs}(t) \leq e^{-2\alpha t}V_{obs}(0) \quad (24)$$

where $\alpha = \lambda_{\min}(\mathbf{Q}) / \lambda_{\max}(\mathbf{P}) > 0$ can be interpreted as the *convergence rate*. The definitions (16) and (17) implies that:

$$\mathbf{K}_1 \triangleq \mathbf{P}_1^{-1}\mathbf{Q}_1 \tag{25}$$

$$\mathbf{K}_2(\mathbf{y}) \triangleq \mathbf{P}_2^{-1}\mathbf{J}^T(\mathbf{y})\mathbf{P}_1 - \mathbf{A}_1 \tag{26}$$

where $\mathbf{P}_2$ is computed from the Lyapunov equation (18) and, $\mathbf{P}_1, \mathbf{Q}_1$ and $\mathbf{Q}_2$ are constant positive definite design matrices.

III. Observer Backstepping

In this section a GES nonlinear control law using the observer in Section II is derived. The observer backstepping problem is solved in two steps by using a *vectorial backstepping* approach. The tracking objective is specified in earth-fixed coordinates by defining a smooth reference trajectory $\boldsymbol{\eta}_d = [x_d, y_d, \psi_d]^T$ satisfying:

$$\ddot{\boldsymbol{\eta}}_d, \dot{\boldsymbol{\eta}}_d, \boldsymbol{\eta}_d \in \mathcal{L}_\infty \tag{27}$$

Station-keeping implies that $\boldsymbol{\eta}_d$ = constant whereas a time-varying $\boldsymbol{\eta}_d$ is specified for tracking control.

Step 1:

Since $\boldsymbol{\eta}$ is measured with sensor noise, the tracking error $\boldsymbol{\eta} - \boldsymbol{\eta}_d$ is rewritten in terms of the estimate of $\boldsymbol{\eta}$. Moreover, the tracking error $\hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_d$ is used for observer backstepping since the observer guarantees that $\hat{\boldsymbol{\eta}} \to \boldsymbol{\eta}$. Hence, the resulting control law will use feedback from the filtered measurements $\hat{\boldsymbol{\eta}}$ instead of $\boldsymbol{\eta}$. Defining the error variable:

$$\mathbf{z}_1 = \hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_d \tag{28}$$

implies that:

$$\dot{\mathbf{z}}_1 = \mathbf{J}(\mathbf{y})\hat{\boldsymbol{\nu}} + \mathbf{K}_1\tilde{\boldsymbol{\eta}} - \dot{\boldsymbol{\eta}}_d \tag{29}$$

The main idea of backstepping is to choose one of the state variables as *virtual control*. It turns out that:

$$\boldsymbol{\xi}_1 = \mathbf{J}(\mathbf{y})\hat{\boldsymbol{\nu}} \triangleq \mathbf{z}_2 + \boldsymbol{\alpha}_1 \tag{30}$$

is an excellent choice for the virtual control. Notice that the virtual control $\boldsymbol{\xi}_1$ is defined as the sum of the next error variable $\mathbf{z}_2$ and $\boldsymbol{\alpha}_1$ which can be interpreted as a *stabilizing function*. Hence:

$$\dot{\mathbf{z}}_1 = \mathbf{z}_2 + \boldsymbol{\alpha}_1 + \mathbf{K}_1\tilde{\boldsymbol{\eta}} - \dot{\boldsymbol{\eta}}_d \tag{31}$$

The *stabilizing function* is chosen as:

$$\boldsymbol{\alpha}_1 = -\mathbf{C}_1\mathbf{z}_1 - \mathbf{D}_1\mathbf{z}_1 + \dot{\boldsymbol{\eta}}_d \tag{32}$$

where $\mathbf{C}_1$ is a strictly positive constant feedback design matrix usually chosen to be diagonal and $\mathbf{D}_1$ is a positive diagonal damping matrix defined according to:

$$\mathbf{D}_1 = \begin{bmatrix} d_1\mathbf{k}_1^T\mathbf{k}_1 & 0 & 0 \\ 0 & d_2\mathbf{k}_2^T\mathbf{k}_2 & 0 \\ 0 & 0 & d_3\mathbf{k}_3^T\mathbf{k}_3 \end{bmatrix} \tag{33}$$

where $d_i > 0$ (i=1...3) and $\mathbf{k}_i$ (i=1...3) are the column vectors of:

$$\mathbf{K}_1^T = [\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3] \tag{34}$$

The motivation for the damping term $-\mathbf{D}_1\mathbf{z}_1$ is that $\mathbf{K}_1\tilde{\boldsymbol{\eta}}$ can be treated as a disturbance term in (29) to be compensated for by adding damping. The final equation for $\dot{\mathbf{z}}_1$ is then:

$$\dot{\mathbf{z}}_1 = -(\mathbf{C}_1 + \mathbf{D}_1)\mathbf{z}_1 + \mathbf{z}_2 + \mathbf{K}_1\tilde{\boldsymbol{\eta}} \tag{35}$$

Notice that $\mathbf{D}_1$ is only used to compensate for the "disturbance" term $\mathbf{K}_1\tilde{\boldsymbol{\eta}}$. In the next step, the desired dynamics of $\mathbf{z}_2$ will be specified.

Step 2:

Time differentiation of (30) yields:

$$\dot{\mathbf{z}}_2 = \dot{\boldsymbol{\xi}}_1 - \dot{\boldsymbol{\alpha}}_1 \tag{36}$$

which can be written according to:

$$\begin{aligned} \dot{\mathbf{z}}_2 &= \mathbf{J}(\mathbf{y})\dot{\hat{\boldsymbol{\nu}}} + \dot{\mathbf{J}}(\mathbf{y})\hat{\boldsymbol{\nu}} + \mathbf{C}_1\dot{\mathbf{z}}_1 + \mathbf{D}_1\dot{\mathbf{z}}_1 - \ddot{\boldsymbol{\eta}}_d \\ &\Updownarrow \\ \dot{\mathbf{z}}_2 &= -(\mathbf{C}_1 + \mathbf{D}_1)^2\mathbf{z}_1 \\ &\quad +(\mathbf{C}_1 + \mathbf{D}_1)(\mathbf{z}_2 + \mathbf{K}_1\tilde{\boldsymbol{\eta}}) - \ddot{\boldsymbol{\eta}}_d + \dot{\mathbf{J}}(\mathbf{y})\hat{\boldsymbol{\nu}} \\ &\quad +\mathbf{J}(\mathbf{y})(\mathbf{A}_1\hat{\boldsymbol{\eta}} + \mathbf{A}_2\hat{\boldsymbol{\nu}} + \mathbf{B}\boldsymbol{\tau} + \mathbf{K}_2\tilde{\boldsymbol{\eta}}) \end{aligned} \tag{37}$$

Defining:

$$\boldsymbol{\rho} = \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix}, \ \mathbf{S}(\boldsymbol{\rho}) = \begin{bmatrix} 0 & -r & 0 \\ r & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{38}$$

and $\tilde{\boldsymbol{\rho}} = \boldsymbol{\rho} - \hat{\boldsymbol{\rho}}$. Hence the time derivative of $\mathbf{J}(\mathbf{y})$ can be written:

$$\dot{\mathbf{J}}(\mathbf{y}) = \mathbf{J}(\mathbf{y})\mathbf{S}(\boldsymbol{\rho}) = \mathbf{J}(\mathbf{y})\mathbf{S}(\tilde{\boldsymbol{\rho}}) + \mathbf{J}(\mathbf{y})\mathbf{S}(\hat{\boldsymbol{\rho}}) \quad (39)$$

Hence the product $\dot{\mathbf{J}}(\mathbf{y})\hat{\boldsymbol{\nu}}$ in (37) can be rewritten as:

$$\begin{aligned} \dot{\mathbf{J}}(\mathbf{y})\hat{\boldsymbol{\nu}} &= \mathbf{J}(\mathbf{y})\mathbf{S}(\tilde{\boldsymbol{\rho}})\hat{\boldsymbol{\nu}} + \mathbf{J}(\mathbf{y})\mathbf{S}(\hat{\boldsymbol{\rho}})\hat{\boldsymbol{\nu}} \\ &= \mathbf{J}(\mathbf{y})\mathbf{T}(\hat{\boldsymbol{\nu}})\tilde{\boldsymbol{\nu}} + \mathbf{J}(\mathbf{y})\mathbf{S}(\hat{\boldsymbol{\rho}})\hat{\boldsymbol{\nu}} \quad (40) \end{aligned}$$

where

$$\mathbf{T}(\hat{\boldsymbol{\nu}}) = \begin{bmatrix} 0 & 0 & -\hat{v} \\ 0 & 0 & \hat{u} \\ 0 & 0 & 0 \end{bmatrix} \quad (41)$$

Substituting (40) into (37) yields:

$$\begin{aligned} \dot{\mathbf{z}}_2 = & -(\mathbf{C}_1 + \mathbf{D}_1)^2\mathbf{z}_1 \\ & +(\mathbf{C}_1 + \mathbf{D}_1)(\mathbf{z}_2 + \mathbf{K}_1\tilde{\boldsymbol{\eta}}) - \ddot{\boldsymbol{\eta}}_d \\ & +\mathbf{J}(\mathbf{y})(\mathbf{A}_1\hat{\boldsymbol{\eta}} + \mathbf{A}_2\hat{\boldsymbol{\nu}} + \mathbf{B}\boldsymbol{\tau} + \mathbf{K}_2\tilde{\boldsymbol{\eta}}) \\ & +\mathbf{J}(\mathbf{y})\mathbf{T}(\hat{\boldsymbol{\nu}})\tilde{\boldsymbol{\nu}} + \mathbf{J}(\mathbf{y})\mathbf{S}(\hat{\boldsymbol{\rho}})\hat{\boldsymbol{\nu}} \quad (42) \end{aligned}$$

Collecting terms in $\tilde{\boldsymbol{\eta}}$ and $\tilde{\boldsymbol{\nu}}$,yields:

$$\begin{aligned} \dot{\mathbf{z}}_2 = & \left((\mathbf{C}_1 + \mathbf{D}_1)\mathbf{K}_1 + \mathbf{J}(\mathbf{y})\mathbf{K}_2\right)\tilde{\boldsymbol{\eta}} \\ & +\mathbf{J}(\mathbf{y})\mathbf{T}(\hat{\boldsymbol{\nu}})\tilde{\boldsymbol{\nu}} \\ & +\boldsymbol{\varphi}(\ddot{\boldsymbol{\eta}}_{\mathbf{d}}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, \hat{\boldsymbol{\rho}}, \mathbf{y}) + \mathbf{J}(\mathbf{y})\mathbf{B}\boldsymbol{\tau} \quad (43) \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{\varphi}(\ddot{\boldsymbol{\eta}}_{\mathbf{d}}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, \hat{\boldsymbol{\rho}}, \mathbf{y}) = & -(\mathbf{C}_1 + \mathbf{D}_1)^2\mathbf{z}_1 \\ & +(\mathbf{C}_1 + \mathbf{D}_1)\mathbf{z}_2 - \ddot{\boldsymbol{\eta}}_d \\ & +\mathbf{J}(\mathbf{y})\mathbf{A}_1\hat{\boldsymbol{\eta}} \quad (44) \\ & +\mathbf{J}(\mathbf{y})(\mathbf{A}_2 + \mathbf{S}(\hat{\boldsymbol{\rho}}))\hat{\boldsymbol{\nu}} \end{aligned}$$

The following choice of feedback is made:

$$\begin{aligned} \boldsymbol{\tau} = & -\mathbf{B}^{-1}\mathbf{J}^T(\mathbf{y})(\boldsymbol{\varphi}(\ddot{\boldsymbol{\eta}}_{\mathbf{d}}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, \hat{\boldsymbol{\rho}}, \mathbf{y}) \\ & +\mathbf{C}_2\mathbf{z}_2 + \mathbf{D}_2\mathbf{z}_2 + \mathbf{z}_1) \quad (45) \end{aligned}$$

where $\mathbf{C}_2$ is a strictly positive feedback design matrix usually chosen to be diagonal. The resulting $\mathbf{z}_2$-error dynamics is:

$$\dot{\mathbf{z}}_2 = -\mathbf{C}_2\mathbf{z}_2 - \mathbf{D}_2\mathbf{z}_2 - \mathbf{z}_1 + \boldsymbol{\Omega}_1\tilde{\boldsymbol{\eta}} + \boldsymbol{\Omega}_2\tilde{\boldsymbol{\nu}} \quad (46)$$

where

$$\boldsymbol{\Omega}_1 \triangleq (\mathbf{C}_1 + \mathbf{D}_1)\mathbf{K}_1 + \mathbf{J}(\mathbf{y})\mathbf{K}_2 \quad (47)$$

$$\boldsymbol{\Omega}_2 \triangleq \mathbf{J}(\mathbf{y})\mathbf{T}(\hat{\boldsymbol{\nu}}) \quad (48)$$

The matrix $\mathbf{D}_2$ is defined in terms of the elements of $\boldsymbol{\Omega}_1$ and $\boldsymbol{\Omega}_2$, that is:

$$\boldsymbol{\Omega}_1^T = [\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_3], \ \boldsymbol{\Omega}_2^T = [\boldsymbol{\omega}_4, \boldsymbol{\omega}_5, \mathbf{0}] \quad (49)$$

with:

$$\begin{aligned} \mathbf{D}_2 = & \operatorname{diag}\{d_4(\boldsymbol{\omega}_1^T\boldsymbol{\omega}_1 + \boldsymbol{\omega}_4^T\boldsymbol{\omega}_4), \\ & d_5(\boldsymbol{\omega}_2^T\boldsymbol{\omega}_2 + \boldsymbol{\omega}_5^T\boldsymbol{\omega}_5), d_6\boldsymbol{\omega}_3^T\boldsymbol{\omega}_3)\} \quad (50) \end{aligned}$$

where $d_i > 0$ (i=4...6).

IV. Stability Analysis

A. Error Dynamics

The resulting error dynamics can be written:

$$\begin{aligned} \dot{\mathbf{z}} &= -(\mathbf{C}_z + \mathbf{D}_z + \mathbf{E})\mathbf{z} + \mathbf{W}_1\tilde{\boldsymbol{\eta}} + \mathbf{W}_2\tilde{\boldsymbol{\nu}} \quad (51) \\ \dot{\tilde{\boldsymbol{\eta}}} &= \mathbf{J}(\mathbf{y})\tilde{\boldsymbol{\nu}} - \mathbf{K}_1\tilde{\boldsymbol{\eta}} \quad (52) \\ \dot{\tilde{\boldsymbol{\nu}}} &= (\mathbf{A}_1 - \mathbf{K}_2)\tilde{\boldsymbol{\eta}} + \mathbf{A}_2\tilde{\boldsymbol{\nu}} \quad (53) \end{aligned}$$

where $\mathbf{z} = [\mathbf{z}_1^T, \mathbf{z}_2^T]^T$ and:

$$\mathbf{C}_z = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_2 \end{bmatrix}, \mathbf{D}_z = \begin{bmatrix} \mathbf{D}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_2 \end{bmatrix} \quad (54)$$

$$\mathbf{E} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{I} & \mathbf{0} \end{bmatrix}, \mathbf{W}_1 = \begin{bmatrix} \mathbf{K}_1 \\ \boldsymbol{\Omega}_1 \end{bmatrix}, \mathbf{W}_2 = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\Omega}_2 \end{bmatrix} \quad (55)$$

B. Lyapunov Stability Analysis

A Lyapunov function candidate for the control law is:

$$V_{con} = \frac{1}{2}\mathbf{z}^T\mathbf{z} > 0, \ \forall \mathbf{z} \neq \mathbf{0} \quad (56)$$

Hence, a Lyapunov function candidate for both the control law and observer can be defined as:

$$\begin{aligned} V &= V_{con} + V_{obs} \\ & \Updownarrow \quad (57) \\ V &= \frac{1}{2}\left(\mathbf{z}^T\mathbf{z} + \tilde{\boldsymbol{\eta}}^T\mathbf{P}_1\tilde{\boldsymbol{\eta}} + \tilde{\boldsymbol{\nu}}^T\mathbf{P}_2\tilde{\boldsymbol{\nu}}\right) \quad (58) \end{aligned}$$

Time differentiation of V along the trajectories of $\mathbf{z}, \tilde{\boldsymbol{\eta}}$ and $\tilde{\boldsymbol{\nu}}$, yields:

$$\dot{V} = \mathbf{z}^T\dot{\mathbf{z}} + \dot{V}_{obs} \tag{59}$$

where the expression for $\dot{V}_{obs}$ is given by (19). Moreover:

$$\dot{V} = \mathbf{z}^T\dot{\mathbf{z}} - \tilde{\boldsymbol{\eta}}^T\mathbf{Q}_1\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{\nu}}^T\mathbf{Q}_2\tilde{\boldsymbol{\nu}} \tag{60}$$

Substituting (51) into (60), and using the fact that $\mathbf{z}^T\mathbf{E}\mathbf{z} = 0$, $\forall \mathbf{z}$, yields:

$$\begin{aligned} \dot{V} &= \mathbf{z}^T\left(-\mathbf{C}_z\mathbf{z} - \mathbf{D}_z\mathbf{z} + \mathbf{W}_1\tilde{\boldsymbol{\eta}} + \mathbf{W}_2\tilde{\boldsymbol{\nu}}\right) \\ &\quad -\tilde{\boldsymbol{\eta}}^T\mathbf{Q}_1\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{\nu}}^T\mathbf{Q}_2\tilde{\boldsymbol{\nu}} \end{aligned} \tag{61}$$

Adding the zero terms:

$$\frac{1}{4}\left(\tilde{\boldsymbol{\eta}}^T\mathbf{G}_1\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{\eta}}^T\mathbf{G}_1\tilde{\boldsymbol{\eta}}\right) = 0 \tag{62}$$

$$\frac{1}{4}\left(\tilde{\boldsymbol{\nu}}^T\mathbf{G}_2\tilde{\boldsymbol{\nu}} - \tilde{\boldsymbol{\nu}}^T\mathbf{G}_2\tilde{\boldsymbol{\nu}}\right) = 0 \tag{63}$$

to (61) yields:

$$\begin{aligned} \dot{V} &= -\mathbf{z}^T\mathbf{C}_z\mathbf{z} - \mathbf{z}^T\mathbf{D}_z\mathbf{z} \\ &\quad +\mathbf{z}^T\mathbf{W}_1\tilde{\boldsymbol{\eta}} + \mathbf{z}^T\mathbf{W}_2\tilde{\boldsymbol{\nu}} \\ &\quad -\frac{1}{4}(\tilde{\boldsymbol{\eta}}^T\mathbf{G}_1\tilde{\boldsymbol{\eta}} + \tilde{\boldsymbol{\nu}}^T\mathbf{G}_2\tilde{\boldsymbol{\nu}}) \\ &\quad -\tilde{\boldsymbol{\eta}}^T(\mathbf{Q}_1 - \frac{1}{4}\mathbf{G}_1)\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{\nu}}^T(\mathbf{Q}_2 - \frac{1}{4}\mathbf{G}_2)\tilde{\boldsymbol{\nu}} \end{aligned} \tag{64}$$

The matrices $\mathbf{G}_1$ and $\mathbf{G}_2$ are defined as:

$$\mathbf{G}_1 \triangleq g_1\mathbf{I}, \qquad \mathbf{G}_2 \triangleq g_2\mathbf{I} \tag{65}$$

where

$$g_1 = \sum_{i=1}^{6}\frac{1}{d_i}, \qquad g_2 = \frac{1}{d_4} + \frac{1}{d_5} \tag{66}$$

In Appendix I it is shown that:

$$\begin{aligned} &-\mathbf{z}^T\mathbf{D}_z\mathbf{z} + \mathbf{z}^T\mathbf{W}_1\tilde{\boldsymbol{\eta}} + \mathbf{z}^T\mathbf{W}_2\tilde{\boldsymbol{\nu}} \\ &\quad -\tfrac{1}{4}(\tilde{\boldsymbol{\eta}}^T\mathbf{G}_1\tilde{\boldsymbol{\eta}} + \tilde{\boldsymbol{\nu}}^T\mathbf{G}_2\tilde{\boldsymbol{\nu}}) \leq 0 \end{aligned} \tag{67}$$

Hence:

$$\dot{V} \leq -\mathbf{z}^T\mathbf{C}_z\mathbf{z} - \tilde{\boldsymbol{\eta}}^T(\mathbf{Q}_1 - \frac{1}{4}\mathbf{G}_1)\tilde{\boldsymbol{\eta}} - \tilde{\boldsymbol{\nu}}^T(\mathbf{Q}_2 - \frac{1}{4}\mathbf{G}_2)\tilde{\boldsymbol{\nu}} \tag{68}$$

It is then clear that $\dot{V}$ can be made negative definite by choosing the positive definite weight matrices $\mathbf{Q}_1 = \mathbf{Q}_1^T$ and $\mathbf{Q}_2 = \mathbf{Q}_2^T$ such that $\|\mathbf{Q}_1\| > g_1/4$ and $\|\mathbf{Q}_2\| > g_2/4$. Notice that $\mathbf{G}_1$ and $\mathbf{G}_2$ are not needed for implementation.

Hence, according to Lyapunov stability theory, the ship model (6)–(7), with control law (44)–(45) and observer (10)–(11) is GES.

C. Resulting Control Law

The resulting control law is given below.

Control law:

$$\boldsymbol{\tau} = -\mathbf{B}^{-1}\mathbf{J}^T(\mathbf{y})(\boldsymbol{\varphi}(\ddot{\boldsymbol{\eta}}_\mathbf{d}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, \hat{\boldsymbol{\rho}}, \mathbf{y}) + \mathbf{C}_2\mathbf{z}_2 + \mathbf{D}_2\mathbf{z}_2 + \mathbf{z}_1)$$

$$\boldsymbol{\varphi}(\ddot{\boldsymbol{\eta}}_\mathbf{d}, \hat{\boldsymbol{\eta}}, \hat{\boldsymbol{\nu}}, \hat{\boldsymbol{\rho}}, \mathbf{y}) = -\ddot{\boldsymbol{\eta}}_d - (\mathbf{C}_1 + \mathbf{D}_1)^2\mathbf{z}_1 + (\mathbf{C}_1 + \mathbf{D}_1)\mathbf{z}_2 + \mathbf{J}(\mathbf{y})\mathbf{A}_1\hat{\boldsymbol{\eta}} + \mathbf{J}(\mathbf{y})(\mathbf{A}_2 + \mathbf{S}(\hat{\boldsymbol{\rho}}))\hat{\boldsymbol{\nu}}$$

$$\mathbf{z}_1 = \hat{\boldsymbol{\eta}} - \boldsymbol{\eta}_d$$

$$\mathbf{z}_2 = \mathbf{J}(\mathbf{y})\hat{\boldsymbol{\nu}} - \boldsymbol{\alpha}_1$$

Stabilizing function:

$$\boldsymbol{\alpha}_1 = -\mathbf{C}_1\mathbf{z}_1 - \mathbf{D}_1\mathbf{z}_1 + \dot{\boldsymbol{\eta}}_d$$

Observer:

$$\dot{\hat{\mathbf{n}}} = \mathbf{J}(\mathbf{y})\hat{\boldsymbol{\nu}} + \mathbf{K}_1\tilde{\boldsymbol{\eta}}$$

$$\dot{\hat{\boldsymbol{\nu}}} = \mathbf{A}_1\hat{\boldsymbol{\eta}} + \mathbf{A}_2\hat{\boldsymbol{\nu}} + \mathbf{B}\boldsymbol{\tau} + \mathbf{K}_2(\mathbf{y})\tilde{\boldsymbol{\eta}}$$

Observer gains:

$$\mathbf{K}_1 \triangleq \mathbf{P}_1^{-1}\mathbf{Q}_1$$

$$\mathbf{K}_2(\mathbf{y}) \triangleq \mathbf{P}_2^{-1}\mathbf{J}^T(\mathbf{y})\mathbf{P}_1 - \mathbf{A}_1$$

$$\tfrac{1}{2}\left(\mathbf{P}_2\mathbf{A}_2 + \mathbf{A}_2^T\mathbf{P}_2\right) \triangleq -\mathbf{Q}_2$$

Nonlinear damping:

$$\boldsymbol{\Omega}_1 = (\mathbf{C}_1 + \mathbf{D}_1)\mathbf{K}_1 + \mathbf{J}(\mathbf{y})\mathbf{K}_2(\mathbf{y})$$

$$\boldsymbol{\Omega}_2 = \mathbf{J}(\mathbf{y})\mathbf{T}(\hat{\boldsymbol{\nu}})$$

$$\boldsymbol{\Omega}_1^T = [\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_3]$$

$$\boldsymbol{\Omega}_2^T = [\boldsymbol{\omega}_4, \boldsymbol{\omega}_5, \mathbf{0}]$$

$$\mathbf{K}_1^T = [\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3]$$

$$\mathbf{D}_1 = \mathrm{diag}\{d_1\mathbf{k}_1^T\mathbf{k}_1, d_2\mathbf{k}_2^T\mathbf{k}_2, d_3\mathbf{k}_3^T\mathbf{k}_3\}$$

$$\mathbf{D}_2 = \mathrm{diag}\{d_4(\boldsymbol{\omega}_1^T\boldsymbol{\omega}_1 + \boldsymbol{\omega}_4^T\boldsymbol{\omega}_4), d_5(\boldsymbol{\omega}_2^T\boldsymbol{\omega}_2 + \boldsymbol{\omega}_5^T\boldsymbol{\omega}_5), d_6\boldsymbol{\omega}_3^T\boldsymbol{\omega}_3\}$$

Design matrices and constants:

- $\mathbf{P}_1$ positive definite
- $\mathbf{Q}_1$ positive definite and $\|\mathbf{Q}_1\| > \frac{g_1}{4}$
- $\mathbf{Q}_2$ positive definite and $\|\mathbf{Q}_2\| > \frac{g_2}{4}$
- $\mathbf{C}_1$ strictly positive
- $\mathbf{C}_2$ strictly positive
- $d_i > 0$ (i=1...6)

V. Case Studies

Two case studies will be presented to illustrate the performance of the proposed controller. In the first one, thruster assisted mooring of a tanker is discussed, see Figure 1. The second case

Figure 1 – Moored tanker $L = 200.6$ (m).

Figure 2 – Offshore supply vessel $L = 76.2$ (m).

study discusses dynamic positioning of a supply vessel, see Figure 2. In both cases the control law and observer parameters are chosen according to:

$$\begin{aligned}
\mathbf{P}_1 &= \text{diag}\{3.0, 3.0, 1.0\} \\
\mathbf{Q}_1 &= \text{diag}\{1.0, 1.0, 1.0\} \\
\mathbf{Q}_2 &= \text{diag}\{1.0, 1.0, 1.0\} \\
\mathbf{C}_1 &= 0.1 \cdot \text{diag}\{1.0, 1.0, 1.0\} \\
\mathbf{C}_2 &= 0.1 \cdot \text{diag}\{1.0, 1.0, 1.0\} \\
d_1 &= d_2 = 10.0, \qquad d_3 = d_4 = d_5 = d_6 = 1.0
\end{aligned}$$

The sampling time is 0.1 (s). In addition to this, Gaussian white noise was added to the measurements and the ship dynamics in order to demonstrate the filtering properties of the observer.

A. Thruster Assisted Mooring of a Tanker

Consider the tanker shown in Figure 1. The *non-dimensional* system matrices describing the moored tanker are given below (*Bis-scaled values* [2]):

$$\mathbf{M} = \begin{bmatrix} 1.0852 & 0 & 0 \\ 0 & 2.0575 & -0.4087 \\ 0 & -0.4087 & 0.2153 \end{bmatrix} \qquad (69)$$

$$\mathbf{D} = \begin{bmatrix} 0.0865 & 0 & 0 \\ 0 & 0.0762 & 0.1510 \\ 0 & 0.0151 & 0.0031 \end{bmatrix} \qquad (70)$$

$$\mathbf{K} = \text{diag}\{0.0389, 0.0266, 0\} \qquad (71)$$

Notice that $K_{33} = 0$ (no mooring moment in yaw). This is a good assumption for the ship shown in Figure 1. The *non-dimensional* eigenvalues of the system matrix:

$$\mathbf{A}_2 = -\mathbf{M}^{-1}\mathbf{D} \qquad (72)$$

are: $\lambda_1 = -0.0797, \lambda_2 = -0.3498$ and $\lambda_3 = 0.0212$. Hence, the tanker is course-unstable since the non-dimensional eigenvalue in yaw is positive. Consequently, the assumption that $\mathbf{A}_2$ must be Hurwitz is violated. This did not have an effect of the performance of the ship mainly due to the stabilizing effect and robustness of the control law. It should be noted that if stabilization is a problem, the non-Hurwitz solution for the observer gain matrices should be applied [8].

The *dimensional* time constants are computed according to:

$$T_i = -\frac{1}{\lambda_i}\sqrt{L/g} \qquad (i = 1...3) \qquad (73)$$

where $L = 200.6$ (m) is the length of the ship hull and $g = 9.81$ (m/s^2). Hence the time constants in *surge, sway* and *yaw* are found to be 56.7, 12.9 and -213.5 (s). The performance of the nonlinear control law is shown in Figures 3 and 4 where the desired yaw angle (heading) command are 10, 5 and 0 (deg). The desired (x, y)-positions are shifted from (-10,10) to (0,0) during the course-changing maneuver. Smooth reference trajectories in surge, sway and yaw are generated by using three 2nd-order low-pass filters with relative damping ratios equal to 1.0 and natural frequencies equal to 0.5 (rad/s). The computer simulations show that the output feedback controller is highly robust for noise contaminated measurements, see Figures 3 and 4.

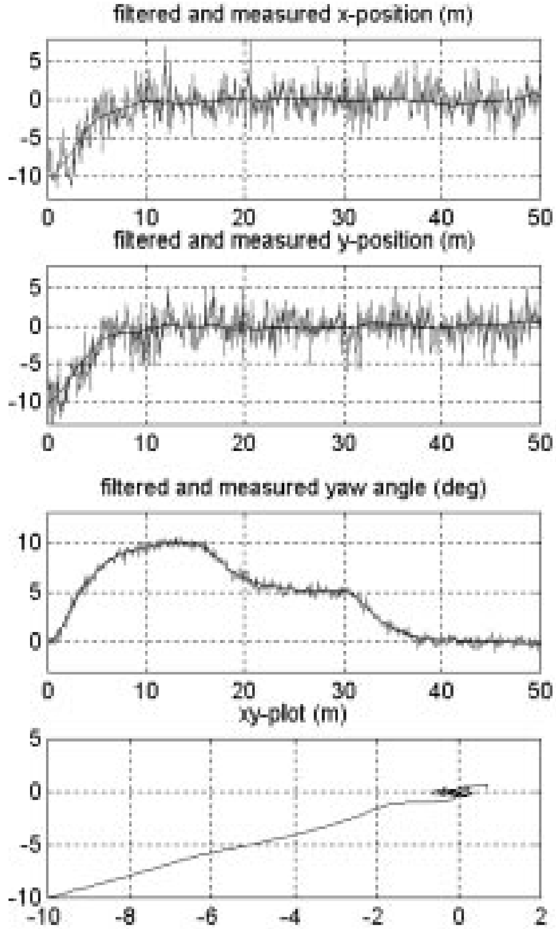

Figure 3 – Thruster assisted mooring of a tanker. The desired positions are changed from (-10,-10) to (0,0). The desired yaw angle is changed between 10, 5 and 0 (deg).

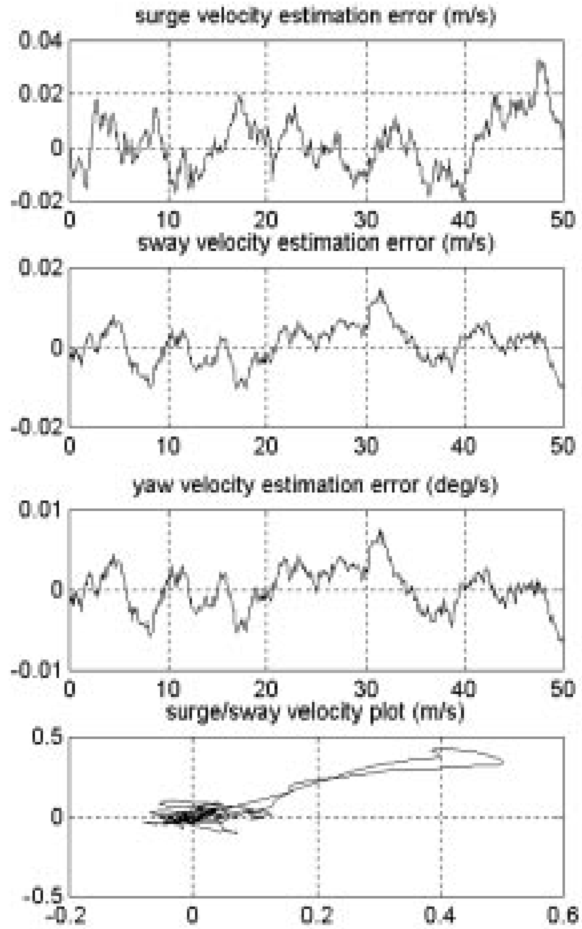

Figure 4 – Thruster assisted mooring of a tanker. The plots show the observer velocity estimation errors in surge, sway and yaw versus time.

B. Dynamic Positioning of a Supply Vessel

The supply vessel shown in Figure 2 is described by the following *non-dimensional* system matrices (*Bis-scaled values* [2]):

$$\mathbf{M} = \begin{bmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.0744 \\ 0 & -0.0744 & 0.1278 \end{bmatrix} \quad (74)$$

$$\mathbf{D} = \begin{bmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0041 & 0.0308 \end{bmatrix} \quad (75)$$

Notice that $\mathbf{K} = \mathbf{0}$ for a supply vessel (no mooring forces). The model parameters of the supply vessel have been identified by performing full-scale sea-trials in the North Sea, see [4] for details. The *non-dimensional* eigenvalues of the system matrix $\mathbf{A}_2 = -\mathbf{M}^{-1}\mathbf{D}$ are $\lambda_1 = -0.2429$, $\lambda_2 = -0.0627$ and $\lambda_2 = -0.0318$. The *dimensional* time constants are computed according to (73) with $L = 76.2$ (m) resulting in $11.5, 44.5$ and 87.8 (s) for surge, sway and yaw, respectively. The performance of the nonlinear control law is shown in Figures 5 and 6 where a time-varying smooth reference trajectory is used in both surge and yaw whereas the desired sway position is zero. Again, excellent performance is demonstrated for noise contaminated position measurements.

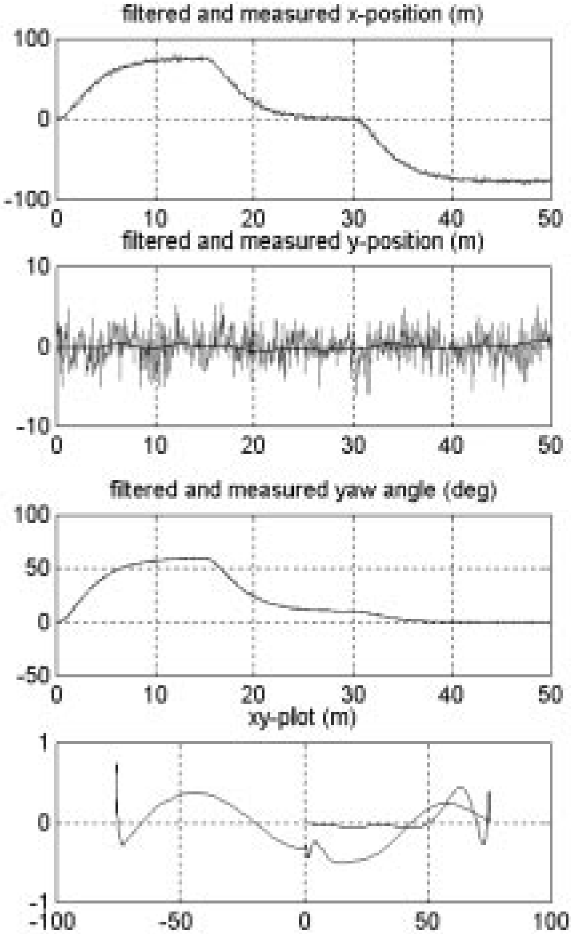

Figure 5 – DP of a supply vessel. Tracking of a time-varying reference trajectory in surge and yaw whereas the desired sway position is zero.

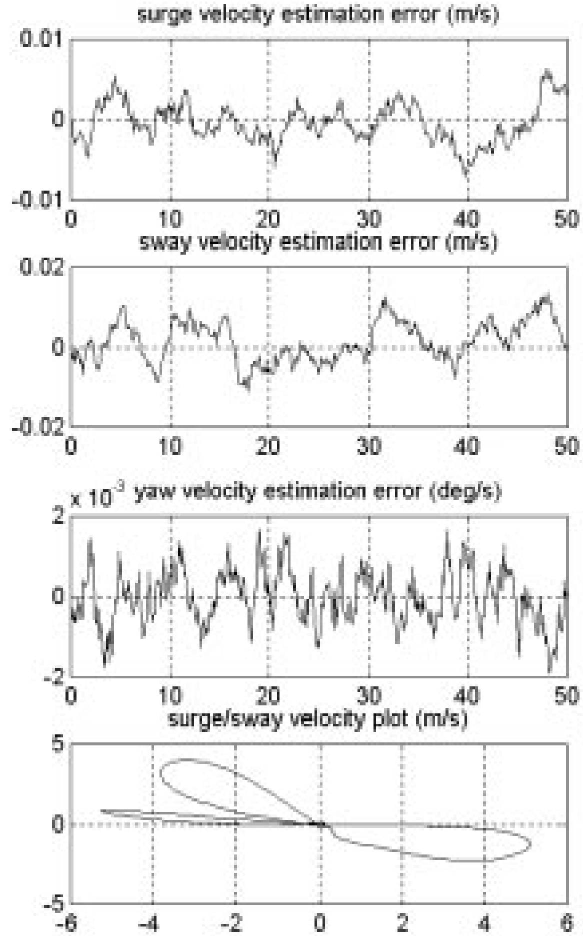

Figure 6 – DP of a supply vessel. The plots show the observer velocity estimation errors in surge, sway and yaw versus time.

VI. Conclusions

A globally exponentially stable (GES) nonlinear output feedback control law for dynamic positioning (DP) and thruster assisted mooring of ships have been derived. The nonlinear control law and observer are based on the nonlinear kinematic equations of motion to describe the position and yaw angle of the vessel. This is advantageous since all conventional DP-systems are designed by linearizing the kinematic equations of motions about a set of predefined constant yaw angles such that linear optimal control theory and gain-scheduling techniques can be applied. The kinematic equations of motion are typically linearized about 36 different yaw angles (steps of 10 degrees to cover 360 degrees). For each of these linearized models, optimal Kalman filter and feedback control gains are computed using the certainty equivalence principle (CEP). The main motivation for this paper has been to show that these assumptions can be removed by using nonlinear observer and feedback control theory. The proposed nonlinear observer is capable of producing noise-free estimates of velocity and position from noisy position measurements. The sensors considered are a gyro compass and a differential global positioning system (DGPS) receiver. Finally, GES of both the observer and control law has been proven by applying vectorial observer backstepping and Lyapunov stability theory. The proposed control law was simulated on a computer by using a mathematical model of an offshore supply vessel and a tanker. The supply vessel was used to demonstrate the excellent tracking capabilities of the output feedback control law whereas the tanker was chosen to demonstrate the applicability towards thruster assisted mooring of tankers.

Appendix I: Proof of Eqn. (68)

Consider (67) which can be expanded according to:

$$\begin{aligned} & -\mathbf{z}^T\mathbf{D}_z\mathbf{z} + \mathbf{z}^T\mathbf{W}_1\bar{\boldsymbol{\eta}} + \mathbf{z}^T\mathbf{W}_2\bar{\boldsymbol{\nu}} \\ & -\frac{1}{4}(\bar{\boldsymbol{\eta}}^T\mathbf{G}_1\bar{\boldsymbol{\eta}} + \bar{\boldsymbol{\nu}}^T\mathbf{G}_2\bar{\boldsymbol{\nu}}) \\ = & -\mathbf{z}_1^T\mathbf{D}_1\mathbf{z}_1 - \mathbf{z}_2^T\mathbf{D}_2\mathbf{z}_2 + \mathbf{z}_1^T\mathbf{K}_1\bar{\boldsymbol{\eta}} + \mathbf{z}_2^T\boldsymbol{\Omega}_1\bar{\boldsymbol{\eta}} \\ & +\mathbf{z}_2^T\boldsymbol{\Omega}_2\bar{\boldsymbol{\nu}} - \frac{g_1}{4}\bar{\boldsymbol{\eta}}^T\bar{\boldsymbol{\eta}} - \frac{g_2}{4}\bar{\boldsymbol{\nu}}^T\bar{\boldsymbol{\nu}} \end{aligned} \quad (76)$$

Equation (76) can be further expanded by using definitions (33), (49) and (50) together with $\mathbf{z}_1 = [z_1, z_2, z_3]^T$ and $\mathbf{z}_2 = [z_4, z_5, z_6]^T$. This yields:

$$\begin{aligned} & -\sum_{i=1}^{3}[d_i(z_i\mathbf{k}_i - \frac{1}{2d_i}\bar{\boldsymbol{\eta}}^T)(z_i\mathbf{k}_i - \frac{1}{2d_i}\bar{\boldsymbol{\eta}}) \\ & +(d_{i+3}z_{i+3}\boldsymbol{\omega}_i - \frac{1}{2d_{i+3}}\bar{\boldsymbol{\eta}})^T(z_{i+3}\boldsymbol{\omega}_i - \frac{1}{2d_{i+3}}\bar{\boldsymbol{\eta}})] \\ & -d_4(z_4\boldsymbol{\omega}_4 - \frac{1}{2d_4}\bar{\boldsymbol{\nu}})^T(z_4\boldsymbol{\omega}_4 - \frac{1}{2d_4}\bar{\boldsymbol{\nu}}) \\ & -d_5(z_5\boldsymbol{\omega}_5 - \frac{1}{2d_5}\bar{\boldsymbol{\nu}})^T(z_5\boldsymbol{\omega}_5 - \frac{1}{2d_5}\bar{\boldsymbol{\nu}}) \end{aligned} \quad (77)$$

Since all the quadratic terms in (77) are less than or equal to zero, (76) and therefore (67) must be negative semi-definite, q.e.d.

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