Controlling Line Tension in Thruster Assisted Mooring Systems

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Abstract— **This paper addresses the potential for energy reduction obtained by using dynamic line tensioning in thruster assisted position mooring systems. Traditionally, mooring systems have been designed in such a way that thruster assistance has not been neccessary under normal environmental conditions. However, as oil production moves into deeper waters, such over-dimensioned mooring systems are no longer feasible. Thus, new "hybrid" solutions must be developed, in which increased thruster action compensates for fewer, and lighter, anchor lines. In this paper, controlling the line tensions dynamically is suggested as an additional means of station keeping, and a control law is derived based on passivity. A model consisting of a rigid-body submodel for the vessel, and a finite element submodel for the mooring system is presented and used for simulations. The simulations show the performance of the proposed control system.**

Keywords— **Ship control, position mooring systems.**

I. Introduction

POSITION mooring systems (PM) have been commercially available since the late 1980's, and have proved to be a cost-effective alternative to permanent platforms for offshore oil production. The current research on PM systems is based on the experience obtained from research on dynamic positioning (DP) systems since the 1970's. DP systems based on optimal control theory and Kalman-filtering were proposed in [1], and extended in [2], [3], [4], [5], [6], [7] and [8]. In recent years, nonlinear controllers have been developed for DP systems based on integrator backstepping techniques (see [9], [10], [11] and [12]).

PM systems differ from DP systems in that thruster assistance is used mainly for damping the surge, sway and yaw motions and for keeping the desired heading, whereas the position is kept within an acceptable region by the mooring lines [13]. Thus, fuel consumption is kept to a minimum in normal weather conditions. In rough weather conditions, thruster assistance may be needed for position keeping in order to avoid line tensions rising above safety limits. In [14], a model for the mooring system based on line characteristics found by solving the catenary equations is presented and the optimal controller derived in [8] is extended for this system. In the last few years, more advanced controllers, based on observer backstepping and locally optimal backstepping (see [15] and [13]) have been developed.

As already mentioned, station keeping is handled entirely by the mooring system under normal environmental conditions. This is possible in shallow waters, where the mooring system can be *over-dimensioned* in such a way that it allows only for small excursions of the vessel from its desired position. In deeper waters, over-dimensioned mooring systems are not feasible, and thruster assistance is needed continuously. This leads us to the motivation for this paper: reduction of fuel consumption. In order to reduce fuel consumption, we suggest using dynamic line tensioning for rejection of constant or slowly-varying disturbances such as mean wind forces, mean currents and tidal currents. Combined with damping in surge, sway and yaw, as well as heading control, this approach is expected to lead to considerable energy savings.

The paper presents a model consisting of a rigid-body submodel for the vessel, and a partial differential equation for the mooring cables. For simulation purposes, spacial discretization of the cable equation is performed using the finite element method. Passivity is shown for the mooring system, from which it is concluded that passive tension controllers, such as the traditional P-, PI- and PID-controllers, ensure stability. Finally, a simulation shows the potential for reduction of energy consumption by using dynamic line tensioning.

II. Modelling

A. Kinematics

Two reference frames are defined as follows (see Figure 1):

1. The Earth-fixed frame, denoted $X_E Y_E Z_E$, is placed so that the origin coincides with the desired position of the vessel. The $X_E Y_E$ plane lies on the water surface, the Z_E axis is positive downwards and the X_E axis points along the desired heading of the vessel.
2. The body-fixed frame, denoted XYZ, is fixed to the vessel body so that the origin coincides with the center of turret. The X axis is directed from aft to fore along the longitudinal axis of the vessel, and the Y axis is directed to starboard.

The vessel position and heading in the Earth-fixed frame are defined by the vector $\eta = [\ x \quad y \quad \psi\ ]^T \in \mathbb{R}^2 \times \mathbb{S}^1$ ($\mathbb{R}^2$ is the Euclidean space of dimension two,

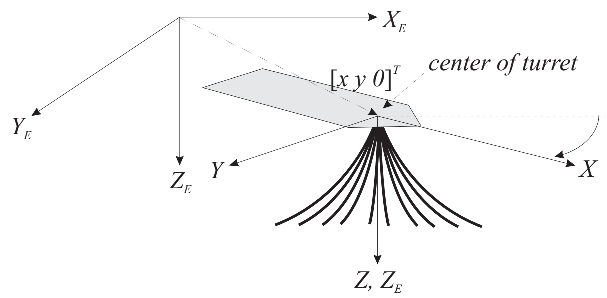

Fig. 1. Earth-fixed ($X_E Y_E Z_E$) and body-fixed (XYZ) reference frames.

and $\mathbb{S}^1$ is the circle). The body-fixed surge, sway and yaw velocities are defined by the vector $\nu = [\ u \quad v \quad r\ ]^T \in \mathbb{R}^3$. The linear velocity of the ship in the Earth-fixed frame $\dot{\eta}$ is related to the velocity in the body-fixed frame ν through a rotation about the Z axis, that is:

$$\mathbf{J}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \in \mathbb{SO}(3)$$

($\mathbb{SO}(3)$ is the proper orthogonal group of transformations on $\mathbb{R}^3$).

B. Vessel dynamics

The low-frequency motion of a free-floating ship in three degrees of freedom, assuming slowly-varying, irrotational fluid flow, can be described by [16]:

$$\mathbf{M}\dot{\nu} + \mathbf{C}(\nu_r)\nu_r + \mathbf{D}\nu_r = \tau$$
$$\dot{\eta} = \mathbf{J}(\psi)\nu$$

where

$$\begin{aligned} \nu_r &= \nu - \mathbf{J}^*(\psi)\ \mathbf{v}_c(\ \mathbf{I}^*\ \eta) \\ \mathbf{J}^*(\psi) &= \mathbf{I}^*\ \mathbf{J}(\psi) \\ \mathbf{I}^* &= \begin{bmatrix} \mathbf{I}_{2\times 2} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix} \end{aligned}$$

$\mathbf{v}_c(z) \in \mathbb{R}^3$ is the current profile in Earth-fixed coordinates, $\mathbf{M}$ is the inertia matrix, $\mathbf{C}$ is the Coriolis and centripetal matrix and $\mathbf{D}$ is the damping matrix. τ constitutes environmental forces (except currents which are already accounted for by defining the relative velocity ν_r), and propulsion forces. In station keeping applications, where the vessel velocity is assumed small, $\mathbf{C}(\nu_r)\nu_r$ is negligible and $\mathbf{D}$ is assumed constant [16].

C. Multi-cable mooring system

The equation of motion of a cable with negligible bending and torsional stiffness, is given by (see for instance [17]):

$$\rho_0 \frac{\partial \mathbf{v}}{\partial t} = \frac{\partial}{\partial s}(T\mathbf{t}) + \mathbf{f}(1+e)$$

where $s \in [0, L]$, $\mathbf{v}(s,t) \in \mathbb{R}^3$ and $\mathbf{t}(s,t) \in \mathbb{R}^3$ are distance along the unstretched cable, velocity and tangential vector, respectively. L is length of the unstretched cable, ρ_0 is mass per unit length of unstretched cable, T is tension, e is strain and $\mathbf{f}(s,t) \in \mathbb{R}^3$ is the sum of external forces (per unit length of unstreched cable) acting on the cable. By introducing the position vector $\mathbf{r}(s) \in \mathbb{R}^3$, and applying *Hooke's law* we get:

$$\rho_0 \frac{\partial^2 \mathbf{r}}{\partial t^2} = \frac{\partial}{\partial s}\left(EA_0 \frac{e}{1+e}\frac{\partial \mathbf{r}}{\partial s}\right) + \mathbf{f}(1+e) \qquad (1)$$

where E is Young's modulus and A_0 is the cross-sectional area of the unstretched cable. The sum of external forces is:

$$\mathbf{f} = \mathbf{f}_{(hg)} + \mathbf{f}_{(dt)} + \mathbf{f}_{(dn)} + \mathbf{f}_{(mn)}$$

where $\mathbf{f}_{(hg)}$ constitutes the bouyancy (gravity and hydrostatic) force per unit length of unstretched cable, $\mathbf{f}_{(td)}$ and $\mathbf{f}_{(nd)}$ are tangential and normal hydrodynamic drag, respectively, per unit length of unstretched cable and $\mathbf{f}_{(mn)}$ is the hydrodynamic inertia force per unit length of unstretched cable. The hydrodynamic forces are modeled by the Morison equation (see for instance [18]). Explicitly the forces are given by:

$$\begin{aligned} \mathbf{f}_{(hg)} &= \rho_0 \frac{\rho_c - \rho_w}{(1+e)\rho_c}\mathbf{g} && (2) \\ \mathbf{f}_{(dt)} &= -\frac{1}{2}C_{DT} d\rho_w \left|\mathbf{v_t}\right| \mathbf{v_t} && (3) \\ \mathbf{f}_{(dn)} &= -\frac{1}{2}C_{DN} d\rho_w \left|\mathbf{v_n}\right| \mathbf{v_n} && (4) \\ \mathbf{f}_{(mn)} &= -C_{MN}\frac{\pi d^2}{4}\rho_w \mathbf{a_n} && (5) \end{aligned}$$

where $\mathbf{g} \in \mathbb{R}^3$ is the gravitational acceleration, ρ_c is density of the cable, ρ_w is density of the ambient water, C_{DT} and C_{DN} are tangential and normal drag coefficients for the cable, respectively, d is the cable diameter, and C_{MN} is a hydrodynamic mass coefficient. $\mathbf{v_t}$ and $\mathbf{v_n}$ are the tangential and normal components of $\mathbf{v}$, respectively, and $\mathbf{a}_n$ is the normal component of $\dot{\mathbf{v}}$.

For the simulation, the following finite element model of a multi-cable mooring system is used. For a complete derivation of the model, see [19]. In this model, each cable of an m-cable mooring system is partitioned into n segments, and the nodal points are enumerated from 0 to n. The position vector in Earth-fixed coordinates for the k^{th} nodal point of the j^{th} cable is denoted $\mathbf{r}_k^j \in \mathbb{R}^3$, and the relative velocity is denoted $\mathbf{v}_k^j = \dot{\mathbf{r}}_k^j - \mathbf{v}_c(\ \mathbf{r}_k^j)$ where $\mathbf{v}_c(\ \mathbf{r}_k^j) \in \mathbb{R}^3$ is the current at $\mathbf{r}_k^j$. The boundary conditions are as follows:

1. The first nodal point $\mathbf{r}_0^j$ is fixed at the bottom for $j = 1, 2, ..., m$.
2. The last nodal point of all cables are connected to each other at the surface, i.e. $\mathbf{r}_n^1 = \mathbf{r}_n^2 = ... = \mathbf{r}_n^m$.

For notational simplicity, we define the following quantities:

$$
\begin{aligned}
\mathbf{r}_{\mathrm{M}} &= \begin{bmatrix} \mathbf{r}_1^{1^T} & \dots & \mathbf{r}_{n-1}^{1^T} & \dots & \mathbf{r}_1^{m^T} & \dots & \mathbf{r}_{n-1}^{m^T} \end{bmatrix}^T \\
\mathbf{v}_{\mathrm{M}} &= \begin{bmatrix} \mathbf{v}_1^{1^T} & \dots & \mathbf{v}_{n-1}^{1^T} & \dots & \mathbf{v}_1^{m^T} & \dots & \mathbf{v}_{n-1}^{m^T} \end{bmatrix}^T \\
\mathbf{k}_{\mathrm{M}} &= \begin{bmatrix} \mathbf{k}_1^{1^T} & \dots & \mathbf{k}_{n-1}^{1^T} & \dots & \mathbf{k}_1^{m^T} & \dots & \mathbf{k}_{n-1}^{m^T} \end{bmatrix}^T \\
\mathbf{g}_{\mathrm{M}} &= \begin{bmatrix} \mathbf{g}_1^{1^T} & \dots & \mathbf{g}_{n-1}^{1^T} & \dots & \mathbf{g}_1^{m^T} & \dots & \mathbf{g}_{n-1}^{m^T} \end{bmatrix}^T \\
\mathbf{M}_{\mathrm{M}} &= \mathrm{diag}\{\mathbf{M}_1^1, \dots, \mathbf{M}_{\mathrm{n}-1}^1, \dots, \mathbf{M}_1^{\mathrm{m}}, \dots, \mathbf{M}_{\mathrm{n}-1}^{\mathrm{m}}\} \\
\mathbf{D}_{\mathrm{M}} &= \mathrm{diag}\{\mathbf{D}_1^1, \dots, \mathbf{D}_{\mathrm{n}-1}^1, \dots, \mathbf{D}_1^{\mathrm{m}}, \dots, \mathbf{D}_{\mathrm{n}-1}^{\mathrm{m}}\} \\
\mathbf{l}_k^j &= \mathbf{r}_k^j - \mathbf{r}_{k-1}^j, \quad \varepsilon_k^j = \left|\mathbf{l}_k^j\right|, \quad \mathbf{P}_k^j = \mathbf{l}_k^j \mathbf{l}_k^{j^T} / \varepsilon_k^{j^2} \\
C_1^j &= C_{MN}^j \frac{\pi d_j^2}{4} \rho_w, \quad C_2^j = \frac{1}{2} C_{DT}^j d_j \rho_w \\
C_3^j &= \frac{1}{2} C_{DN}^j d_j \rho_w
\end{aligned}
$$

where C_{DT}^j and C_{DN}^j are tangential and normal drag coefficients for the j^{th} cable, respectively, C_{MN}^j is a hydrodynamic mass coefficient for the j^{th} cable, l_j is the length of each element in the j^{th} cable and

$$
\begin{aligned}
\mathbf{M}_k^j &= \left[\rho_o l_j + \frac{C_1^j}{2}(\varepsilon_k^j + \varepsilon_{k+1}^j)\right] \mathbf{I}_{3\times 3} - \\
&\quad \frac{C_1^j}{2} \left(\varepsilon_k^j \mathbf{P}_k^j + \varepsilon_{k+1}^j \mathbf{P}_{k+1}^j\right) \\
\mathbf{D}_k^j &= \frac{C_2}{2} \left[\left|\mathbf{v}_k^j \cdot \mathbf{l}_k^j\right| \mathbf{P}_k^j + \left|\mathbf{v}_k^j \cdot \mathbf{l}_{k+1}^j\right| \mathbf{P}_{k+1}^j\right] + \\
&\quad \frac{C_3}{2} \left[\varepsilon_k^j \left|(\mathbf{I}_{3\times 3} - \mathbf{P}_k^j)\mathbf{v}_k^j\right| (\mathbf{I}_{3\times 3} - \mathbf{P}_k^j) + \right. \\
&\quad \left. \varepsilon_{k+1}^j \left|(\mathbf{I}_{3\times 3} - \mathbf{P}_{k+1}^j)\mathbf{v}_k^j\right| (\mathbf{I}_{3\times 3} - \mathbf{P}_{k+1}^j)\right] \\
\mathbf{k}_k^j &= \frac{E_j A_{0_j}}{l_j} \left[\frac{\varepsilon_k^j - l_j}{\varepsilon_k^j} \mathbf{l}_k^j - \frac{\varepsilon_{k+1}^j - l_j}{\varepsilon_{k+1}^j} \mathbf{l}_{k+1}^j\right] \\
\mathbf{g}_k^j &= -l_j \rho_0^j \frac{\rho_c^j - \rho_w}{\rho_c^j} \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T
\end{aligned}
$$

The mooring dynamics takes the form:

$$\mathbf{M}_0 \dot{\mathbf{v}}_0 + \mathbf{D}_0 \mathbf{v}_0 + \mathbf{k}_0 + \mathbf{g}_0 = \mathbf{0}$$

where

$$
\begin{aligned}
\mathbf{r}_0 &= \begin{bmatrix} \mathbf{r}_{\mathrm{M}} \\ \mathbf{r}_n \end{bmatrix}, \quad \mathbf{v}_0 = \begin{bmatrix} \mathbf{v}_{\mathrm{M}} \\ \mathbf{v}_n \end{bmatrix} \\
\mathbf{k}_0 &= \begin{bmatrix} \mathbf{k}_{\mathrm{M}} \\ \mathbf{k}_n \end{bmatrix}, \quad \mathbf{g}_0 = \begin{bmatrix} \mathbf{g}_{\mathrm{M}} \\ \mathbf{g}_n \end{bmatrix} \\
\mathbf{M}_0 &= \begin{bmatrix} \mathbf{M}_{\mathrm{M}} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_n \end{bmatrix}, \quad \mathbf{D}_0 = \begin{bmatrix} \mathbf{D}_{\mathrm{M}} & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_n \end{bmatrix}
\end{aligned}
$$

and

$$
\begin{aligned}
\mathbf{M}_n &= \frac{1}{2} \sum_{j=1}^{m} \left[(\rho_o l_j + C_1^j \varepsilon_n^j) \mathbf{I}_{3\times 3} - C_1^j \varepsilon_n^j \mathbf{P}_n^j\right] \\
\mathbf{D}_n &= \frac{1}{2} \sum_{j=1}^{m} \left[C_2^j \left|\mathbf{v}_n^j \cdot \mathbf{l}_n^j\right| \mathbf{P}_n^j + \right. \\
&\quad \left. C_3^j \varepsilon_n^j \left|(\mathbf{I}_{3\times 3} - \mathbf{P}_n^j)\mathbf{v}_n^j\right| (\mathbf{I}_{3\times 3} - \mathbf{P}_n^j)\right] \\
\mathbf{k}_n &= \sum_{j=1}^{m} \left[\frac{E_j A_{0_j}}{l_j} \frac{\varepsilon_n^j - l_j}{\varepsilon_n^j} \mathbf{l}_n^j\right] \\
\mathbf{g}_n &= -\frac{1}{2} \sum_{j=1}^{m} l_j \rho_0^j \frac{\rho_c^j - \rho_w}{\rho_c^j} \begin{bmatrix} 0 & 0 & g \end{bmatrix}^T
\end{aligned}
$$

D. Coupled vessel-mooring dynamics

The vessel dynamics is described in three degrees of freedom; displacement and rotation in the plane of the sea surface. Thus, when connecting the last node of all cables to the center of turret on the vessel, the following additional boundary conditions are inflicted on the mooring system:

$$\mathbf{r}_n = \begin{bmatrix} x & y & 0 \end{bmatrix}^T = \begin{bmatrix} \mathbf{I}_{2\times 2} & 0 \\ 0 & 0 \end{bmatrix} \eta \tag{6}$$

$$\mathbf{v}_n = \begin{bmatrix} \mathbf{I}_{2\times 2} & 0 \\ 0 & 0 \end{bmatrix} \dot{\eta} - \mathbf{v}_c(\mathbf{I}^* \eta) = \mathbf{J}^*(\psi) \nu_r \tag{7}$$

It is convenient to note that the inertia and drag matrices for the vessel completely dominate those of the upper element of the mooring system. Therefore, the terms involving $\mathbf{M}_n$ and $\mathbf{D}_n$ can be neglected, resulting in the following complete equations of motion for the moored vessel:

$$\mathbf{M}\dot{\nu} + \mathbf{D}\nu_r + \mathbf{J}^{*^T}(\psi) \mathbf{k}_n(\mathbf{r}_0) = \tau \tag{8}$$

$$\mathbf{M}_{\mathrm{M}}(\mathbf{r}_0)\dot{\mathbf{v}}_0 + \mathbf{D}_{\mathrm{M}}(\mathbf{v}_0, \mathbf{r}_0)\mathbf{v}_0 + \mathbf{k}_{\mathrm{M}}(\mathbf{r}_0) + \mathbf{g}_{\mathrm{M}} = \mathbf{0} \tag{9}$$

$$\dot{\eta} = \mathbf{J}(\psi)\nu, \quad \dot{\mathbf{r}}_0 = \mathbf{v}_0 \tag{10}$$

III. Controller design

The design of a controller for dynamic line tensioning is based on the following two basic assumptions:

1. A DP system has already been designed for the vessel.
2. The mooring line tensions are measured continuously.

Denoting the commanded force from the DP controller τ_c, and the mooring force projected onto the horizontal plane τ_m, we define the thruster force (see Figure 2):

$$\tau_{thr} = \tau_c - \tau_m$$

Thus, the performance of the thruster assisted mooring system is exactly that of the DP system alone. The objective of the dynamic line tensioning controller is to track τ_c, such that as little as possible of the commanded force is passed on to the thrusters. Formally,

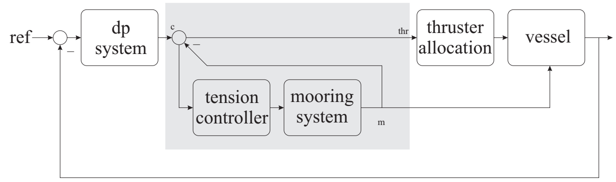

Fig. 2. Block diagram of the moored vessel, with the dynamic line tensioning system emphasized.

the control problem is:

Problem 1:

$$\begin{aligned}
\rho_0 \frac{\partial \mathbf{v}}{\partial t} &= \frac{\partial}{\partial s}\left(EA_0 \frac{e}{1+e}\frac{\partial \mathbf{r}}{\partial s}\right) + \\
&\left(\mathbf{f}_{(hg)} + \mathbf{f}_{(dt)} + \mathbf{f}_{(dn)} + \mathbf{f}_{(mn)}\right)(1+e), \quad s \in [0, L] \\
\mathbf{v}(s) &= \mathbf{v}(L), \quad e(s) = e(L), \quad s \in (L, L_{\max}] \\
\mathbf{r}(0) &= \mathbf{r}_0, \quad \mathbf{r}(L) = \mathbf{0} \\
\mathbf{v}(L) &= [1 + e(L)]\, \dot{L}\mathbf{t}(L) = u\mathbf{t}(L) \\
\frac{dL}{dt} &= \frac{u}{1+e(L)} \\
y &= EA_0 e(L) = T(L) \qquad (11)
\end{aligned}$$

Find $u(t)$ such that $y(t)$ regulates to T_{ref}. ♦

In the problem formulation above, $\mathbf{r}(L) = \mathbf{0}$, which is to say that the DP-system is assumed to be perfect in the analysis. Deviation from the desired vessel position is considered noise.

Criterion 1 (Well Posedness) T_{ref} lies within a certain interval given by the static line characteristic. Formally; there exists $L^* \in [L_{min}, L_{max}]$, $\mathbf{r}^*(s)$ (and thereby $e^*(s)$) such that

$$\frac{\partial}{\partial s}\left(EA_0 \frac{e^*}{1+e^*}\frac{\partial \mathbf{r}^*}{\partial s}\right) + \mathbf{f}_{(hg)}(1+e^*) = 0, \quad s \in [0, L^*]$$

$$\mathbf{r}^*(0) = \mathbf{r}_0, \quad \mathbf{r}^*(L^*) = \mathbf{0}$$

is satisfied, and $T_{ref} = EA_0 e^*(L^*)$. ♦

Moving the added mass term in equation (11) to the left hand side yields:

$$\mathbf{M}_c \begin{bmatrix} \dot{\mathbf{v}}_t \\ \dot{\mathbf{v}}_n \end{bmatrix} = \frac{\partial}{\partial s}(EA_0 \frac{e}{1+e}\frac{\partial \mathbf{r}}{\partial s}) + (\mathbf{f}_{(hg)} + \mathbf{f}_{(dt)} + \mathbf{f}_{(dn)})(1+e)$$

where

$$\mathbf{M}_c = \begin{bmatrix} \rho_0 & 0 \\ 0 & \rho_0 + C_{MN}\frac{\pi d^2}{4}\rho_w(1+e) \end{bmatrix}$$

Assumption 1: The inertia matrix including hydrodynamic added inertia is constant, that is: $\dot{\mathbf{M}}_c = \mathbf{0}$. ♦

Proposition 1: Under Assumption 1, the system (11) is (state strictly) passive.

Proof: Take the storage function

$$V(\mathbf{v}, e) = \frac{1}{2}\int_0^{L_{\max}} \mathbf{v}^T \mathbf{M}_c \mathbf{v}\, ds + \frac{1}{2} EA_0 \int_0^{L_{\max}} e^2 ds$$

The derivative of V with respect to time is:

$$\begin{aligned}
\dot{V} = &\int_0^{L(t)} \mathbf{v} \cdot \frac{\partial}{\partial s}(EA_0 \frac{e}{1+e}\frac{\partial \mathbf{r}}{\partial s}) ds + \\
&\int_0^{L(t)} \mathbf{v} \cdot \mathbf{f}_{(hg)}(1+e) ds + \\
&\int_{L(t)}^{L_{\max}} \mathbf{v}^T(s)\mathbf{M}_c(L)\dot{\mathbf{v}}(L) ds - \\
&\frac{1}{2} d\rho_w \int_0^{L(t)} \left(C_{DT}\left|\mathbf{v}_t\right|^3 + C_{DN}\left|\mathbf{v}_n\right|^3 \right)(1+e) ds + \\
&\frac{1}{2} EA_0 \int_0^{L(t)} \frac{de^2}{dt} ds + \frac{1}{2} EA_0 \int_{L(t)}^{L_{\max}} \frac{de(L)^2}{dt} ds \qquad (12)
\end{aligned}$$

Consider the first term in equation (12):

$$\int_0^{L(t)} \mathbf{v} \cdot \frac{\partial}{\partial s}(EA_0 \frac{e}{1+e}\frac{\partial \mathbf{r}}{\partial s}) ds$$

$$\begin{aligned}
&= \left[\mathbf{v} \cdot EA_0 \frac{e}{1+e}\frac{\partial \mathbf{r}}{\partial s}\right]_0^L - \int_0^{L(t)} \frac{\partial \mathbf{v}}{\partial s} \cdot EA_0 \frac{e}{1+e}\frac{\partial \mathbf{r}}{\partial s} ds \\
&= uy - \int_0^{L(t)} \frac{d\left(\frac{\partial \mathbf{r}}{\partial s}\right)}{dt} \cdot EA_0 \frac{e}{1+e}\frac{\partial \mathbf{r}}{\partial s} ds \\
&= uy - \int_0^{L(t)} \frac{d\left((1+e)\mathbf{t}\right)}{dt} EA_0 e \mathbf{t}\, ds \\
&= uy - EA_0 \int_0^{L(t)} \frac{d\left(e\mathbf{t}\right)}{dt} e\mathbf{t}\, ds \\
&= uy - \frac{1}{2} EA_0 \int_0^{L(t)} \frac{de^2}{dt} ds \qquad (13)
\end{aligned}$$

Now, consider the second term in equation (12):

$$\int_0^{L(t)} \mathbf{v}\cdot\mathbf{f}_{(hg)}(1+e)ds$$

$$= \rho_0 \frac{\rho_c-\rho_w}{\rho_c}\left(\frac{d}{dt}\int_0^{L(t)} \mathbf{r}\cdot\mathbf{g}ds - \mathbf{r}(L)\cdot\mathbf{g}\dot{L}\right) = 0 \quad (14)$$

Now, consider the third term in equation (12):

$$\int_{L(t)}^{L_{\max}} \mathbf{v}^T(s)\mathbf{M}_c(L)\dot{\mathbf{v}}(L)ds$$

$$= \left[\mathbf{v}(s)\cdot EA_0\frac{e(L)}{1+e(L)}\frac{\partial \mathbf{r}}{\partial s}(L)\right]_{L(t)}^{L_{\max}} - \frac{1}{2}EA_0\int_{L(t)}^{L_{\max}}\frac{de(L)^2}{dt}ds$$

$$= -\frac{1}{2}EA_0\int_{L(t)}^{L_{\max}}\frac{de(L)^2}{dt}ds \quad (15)$$

Inserting (13), (14) and (15) into (12) yields:

$$uy \geq \dot{V}(\mathbf{v},e)+\rho\psi(\mathbf{v})$$

where $\rho\in[0,1]$ and

$$\psi(\mathbf{v}) = \frac{1}{2}d\rho_w\int_0^{L(t)}\left(C_{DT}\left|\mathbf{v}_t\right|^3 + C_{DN}\left|\mathbf{v}_n\right|^3\right)(1+e)ds$$

■

Therefore, a passive controller will ensure stability. For dynamically positioned and moored vessels the inertia matrix will be slowly-varying compared to the dynamics of the closed-loop system. Hence, time scale separation suggests that $\dot{\mathbf{M}}_\mathbf{c}=\mathbf{0}$ is a good approximation (Assumption 1). In general, the inertia matrix will depend on the frequency of the incoming waves, speed of the vessel and strain. These effects are, however, negligible in an industrial control system [16].

IV. Simulations

A simple simulation has been performed in order to demonstrate the potential for energy reduction when using dynamic line tensioning. The simulation is simple in the sense that tidal current is the only disturbance considered. Table I summarizes the parameters used. The DP-system described in [20, Section 2.2.3] is used, giving the commanded force τ_c to the tension controller. The mooring-system consists of $m=4$ cables, with anchor points distributed evenly on a circle of radius $2km$ about the origin. The commanded force is distributed to the cables in such a way that two cables having their anchor points opposite each other with respect to the origin are winded, respectively unwinded, at the same speed. This strategy leads to the following P-controller:

TABLE I

Parameters used in the simulation

$L_j = 2250$, $C_{DT}^j = 0.3$, $C_{DN}^j = 1$, $C_{MN}^j = 1$, $\rho_c^j = 5500$, $\rho_0^j = 27.6$, $d_j = 0.08$, $E_jA_{0_j} = 230\times10^6$	$j\in[1,2,3,4]$
$\mathbf{M} = \begin{bmatrix} 9.6\times10^7 & 0 & 0 \\ 0 & 1.3\times10^8 & -5.3\times10^9 \\ 0 & -5.3\times10^9 & 5\times10^{11}\end{bmatrix}$	
$\mathbf{D} = \begin{bmatrix} 9.2\times10^5 & 0 & 0 \\ 0 & 2.4\times10^6 & -9.7\times10^7 \\ 0 & -9.7\times10^7 & 1.3\times10^{10}\end{bmatrix}$	
$\mathbf{v}_c(z,t) = \begin{bmatrix} -\cos(t/6875) \\ \sin(t/6875) \\ 0\end{bmatrix}$	$z\in[0,1000]$
$\rho_w = 1025$	
$\mathbf{r}_0^1 = \begin{bmatrix} 2000 & 0 & 1000\end{bmatrix}^T$	
$\mathbf{r}_0^2 = \begin{bmatrix} 0 & 2000 & 1000\end{bmatrix}^T$	
$\mathbf{r}_0^3 = \begin{bmatrix} -2000 & 0 & 1000\end{bmatrix}^T$	
$\mathbf{r}_0^4 = \begin{bmatrix} 0 & -2000 & 1000\end{bmatrix}^T$	
$k_c = 2.5\times10^{-7}$	

$$\mathbf{u} = -k_c\mathbf{B}(\tau_c-\tau_m)$$

where k_c is the controller gain, τ_c is the commanded force and τ_m is the measured mooring force. $\mathbf{B}$ is an $m\times3$ configuration matrix given by the anchor points:

$$\mathbf{B} = \begin{bmatrix}\mathbf{p}_1 & \mathbf{p}_2 & \cdots & \mathbf{p}_m\end{bmatrix}^T, \quad \mathbf{p}_j = \frac{\mathbf{I}^*\mathbf{r}_0^j}{\left|\mathbf{I}^*\mathbf{r}_0^j\right|}$$

Figure 3 shows commanded force from the DP-system compared with the thruster force. As expected, the thruster force is very small, which means that the tension controller follows the commanded force quite closely. Figure 4 shows the length of the cables with anchor points on the positive x-axis and positive y-axis. The length of the remaining two cables are in exact opposite phase to the ones shown.

V. Conclusions

By showing that the mapping from winding velocity of the cable to the upper end tension is passive, we con-

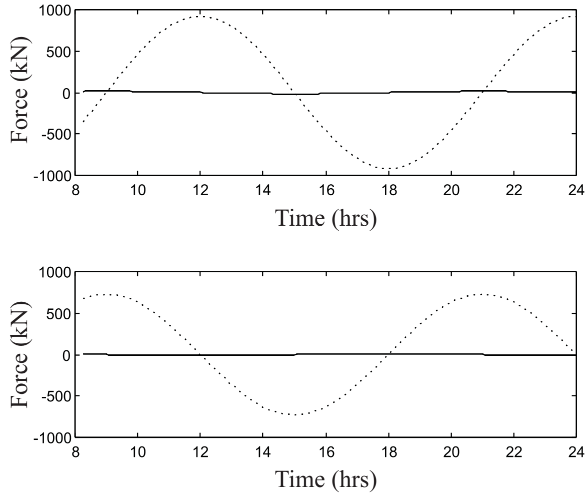

Fig. 3. Commanded (dotted) and thruster forces (solid) in the x-direction (upper graph) and y-direction (lower graph).

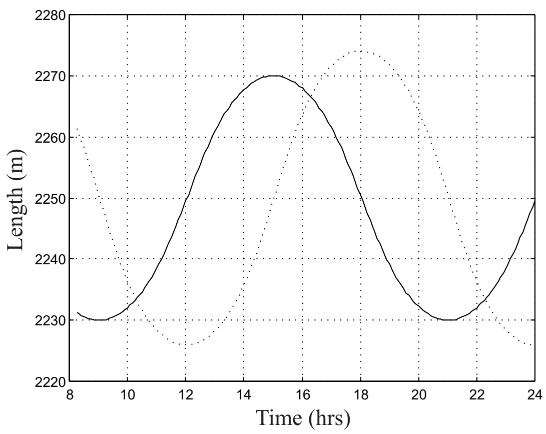

Fig. 4. Length of cables with anchor point at positive x-axis (dotted) and positive y-axis (solid).

clude that passive controllers, such as the traditional P-, PI- and PID-controllers, may be used for dynamic line tensioning in a mooring system. Computer simulations demonstrate the potential for lowering fuel consumption by letting the mooring system compensate for constant and slowly varying environmental forces. Compensation of fast disturbances is still left to the thrusters, as tear and wear on the mooring cables will be a limitation on the action allowed from the tension controller.

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References

[1] J. G. Balchen, N. A. Jenssen, and S. Sælid, "Dynamic positioning using kalman filtering and optimal control theory," in *Proc. of the IFAC/IFIP symposium*, (Amsterdam, The Netherlands.), 1976.

[2] J. G. Balchen, N. A. Jenssen, E. Mathisen, and S. Sælid, "A dynamic positioning system based on kalman filtering and optimal control," *Modeling, Identification and Control (MIC)*, vol. 1, no. 3, pp. 135–163, 1980.

[3] J. G. Balchen, N. A. Jenssen, and S. Sælid, "Dynamic positioning of floating vessels based on kalman filtering and optimal control," in *Proc. of the 19th IEEE Conf. on Decision and Control*, (Albuquerque, NM.), pp. 852–864, 1980.

[4] M. J. Grimble, R. J. Patton, and D. A. Wise, "The design of dynamic ship positioning control systems using stochastic optimal control theory," *Optimal Control Applications and Methods*, pp. 167–202, 1980.

[5] M. J. Grimble, R. J. Patton, and D. A. Wise, "Use of kalman filtering techniques in dynamic ship positioning systems," in *IEE proceedings, Control theory and applications*, pp. 93–102, 1980.

[6] P. T. Fung and M. J. Grimble, "Dynamic ship positioning using self-tuning kalman filter," *IEEE Transactions on Automatic Control*, vol. 28, no. 3, pp. 339–349, 1983.

[7] S. Saelid, N. A. Jenssen, and J. G. Balchen, "Design and analysis of a dynamic positioning system based on kalman filtering and optimal control," *IEEE Transactions on Automatic Control*, vol. 28, no. 3, pp. 331–339, 1983.

[8] A. J. Sørensen, S. I. Sagatun, and T. I. Fossen, "Design of a dynamic positioning system using model-based control," *Journal of Control Engineering Practice*, vol. 4, no. 3, 1996.

[9] T. I. Fossen and Å. Grøvlen, "Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping," *IEEE Transactions on Control Systems Technology*, vol. 6, no. 1, pp. 121–128, 1998.

[10] A. Robertsson and R. Johansson, "Comments on: Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping," *To appear in IEEE Transactions on Control Systems Technology.*

[11] T. I. Fossen and J. P. Strand, "Passive nonlinear observer design for ships using lyapunov methods: Full-scale experiments with a sypply vessel," *Automatica*, vol. 35, no. 1, pp. 3–16, 1999.

[12] M. F. Aarset, J. P. Strand, and T. I. Fossen, "Nonlinear vectorial observer backstepping with integral action and wave filtering for ships," in *Proc. of the IFAC Conf. on Control Applications in Marine Systems (CAMS'98)*, (Fukuoka, Japan), 1998.

[13] J. P. Strand, *Nonlinear Position Control Systems Design for Marine Vessels.* PhD thesis, Dept. of Eng. Cybernetics, Norwegian Univ. of Sci. and Tech., June 1999.

[14] J. P. Strand, A. J. Sørensen, and T. I. Fossen, "Modelling and control of thruster assisted position mooring systems for ships," in *Proc. of the IFAC Conf. on Manouvering and Control of Marine Craft*, pp. 160–165, IFAC, 1997.

[15] J. P. Strand, K. Ezal, T. I. Fossen, and P. V. Kokotović, "Nonlinear control of ships: A locally optimal design," in *Proc. of the IFAC NOLCOS'98*, (Enschede, The Netherlands), 1998.

[16] T. I. Fossen, *Guidance and Control of Ocean Vehicles.* John Wiley & Sons, Inc., 1994.

[17] M. S. Triantafyllou, "Cable mechanics with marine applications, lecture notes," Department of Ocean Engineering, Massachusetts Institute of Technology, Cambridge, MA 02139, USA, May 1990.

[18] O. M. Faltinsen, *Sea Loads on Ships and Offshore Structures.* Cambridge University Press, 1990.

[19] O. M. Aamo and T. I. Fossen, "Finite element modelling of moored vessels," *Submitted to the Journal of Mathematical and Computer Modelling of Dynamical Systems*, 1999.

[20] T. I. Fossen and J. P. Strand, "Nonlinear ship control," Tech. Rep. 98-19-W, Dept. of Eng. Cybernetics, Norwegian Univ. of Sci. and Tech., 1998.