Nonlinear Analysis of GPS Aided by INS

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BIOGRAPHIES

Bjørnar Vik received his MSc degreee in Engineering Cybernetics from the Norwegian University of Technology and Science in 1994, and is currently pursuing his Ph.D. at the same university. His research interests include GPS and INS technology, nonlinear control theory especially within marine applications.

Thor I. Fossen received the M.Sc. degree in Naval Architecture in 1987 from the Norwegian University of Science and Technology (NTNU), Trondheim and the Ph.D. degree in Engineering Cybernetics from NTNU in 1991. In the period 1989-1990 Fossen pursued postgraduate studies as a Fulbright scholar in flight control at the Department of Aeronautics and Astronautics, University of Washington, Seattle. In May 1993 Fossen was appointed as a Professor in Guidance, Navigation and Control at NTNU where he is teaching ship and ROV control systems design, flight control, and nonlinear and adaptive control theory. Fossen is a Senior Scientific Advisor for ABB Industri AS, Marine Division in Oslo where he has worked on ship control. Fossen is the designer of the SeaLaunch trim and heel correction systems which is an offshore rig build by Boeing-Energia-Kværner from which rockets can be launched. Fossen is the author of the book Guidance and Control of Ocean Vehicles (Wiley, 1994) and the coeditor of the book New Directions in Nonlinear Observer Design (Springer-Verlag London, 1999).

ABSTRACT

The advantages of integrating GPS and INS are well known. One of the benefits is that in the event of jamming or high dynamics, the GPS correlator loops can be aided by corrected INS measurements. The external INS measurement allows the bandwidth of the tracking loops to be lowered, thus making it possible for the receiver to track weak satellite signals. The aiding creates an additional feedback loop that is potentially destabilizing, and careful tuning of both the loop filters and the GPS/INS integration filter is necessary. When analyzing tracking loops and GPS/INS integration filters, it is usually assumed that the tracking/estimation errors are small so that linear models and linear analysis can be applied. In this paper we present a nonlinear observer for tight integration of GPS and INS that has been designed using nonlinear system design methodologies. This observer estimates IMU and GPS errors directly within the nonlinear strapdown equations. The corrected INS measurements are used to aid the GPS carrier loops which also are represented with nonlinear models. The amount of feedback coming from the correlator loop output, via the integration filter, and back into the loop, is dependent from the tuning of the integration filter. It is therefore evident that the integration filter and the correlator loops should be tuned to eachother. In this paper we use the concept of Integral Quadratic Constraints (IQCs) to derive conditions for stability. First the stability of a nonlinear third order carrier loop is analyzed using IQCs, and conditions for stability of the system are given. Furthermore, we look at the interconnected GPS/INS system using IQCs. Finally, simulations that show the difference between the linear and nonlinear stability conditions in the first case, will be given.

1 INTRODUCTION

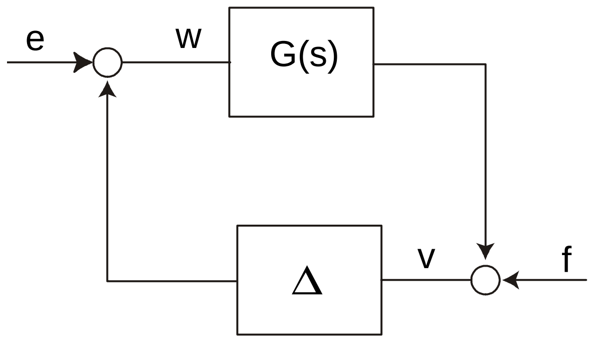
Integration of GPS and INS offers many advantages over stand-alone systems. The long term accuracy of GPS can be combined with the high output rate, high dynamic capability, robustness, and reliability of INS. Also, in the case of GPS dropout the INS will be calibrated when coasting through the outage. After the outage, the INS can provide an estimation of position and velocity to the GPS receiver to speed up reaquisition. Several levels of integration have been defined (see e.g. [1]) , among them uncoupled integration, loose integration, tight integration , and deep integration. Performance will increase with tighter integration at the cost of increased complexity, lack of redundancy and reduced flexibility. One of the benefits introduced with tighter integration, is the possibility of aiding the GPS correlator loops during high levels of stress due to RF interference, jamming, or highly dynamic motion. The aiding enables the GPS correlator loops to lower the bandwidth and thereby reducing the tracking threshold. However, there is a risk of instability when GPS is used for calibration of INS and this calibrated signal is fed back into the GPS receiver. The total system behaviour depend on both the tuning of the integration filter and the GPS correlator loops.

The discriminator chararacteristics of code and carrier loops are in general nonlinear and change as the (C/N_0) levels change. This means that a system tuned to optimal performance using linear analysis will not perform optimally when loop errors are large. The system can even be unstable when it should be stable according to linear analysis. In this paper we want to find conditions for stability that lead to a maximum allowed loop error. This will also give information about tuning when the loop error is large. Two cases will be considered. The unaided carrier loop with a third order loop transfer function, and an aided carrier loop with a second order loop transfer function.

Nonlinear systems theory have been substantially developed the last few decades, but most results are still very conservative, offering only sufficient but not necessary conditions for stability. Also, when combining nonlinear with linear systems, it is difficult to take advantage of the sufficient and necessary conditions that linear analysis give. In this paper we will use the concept of Integral Quadratic Constraints (IQC), developed by V. A. Yakubovich ([2]), to analyze the two cases.

2 INTEGRAL QUADRATIC CONSTRAINTS

An IQC provide a way of representing relationships between processes evolving in a complex system, in a form that is convenient for analysis. depending on the particular application, various versions of IQC's are available.



1. Basic feedback system.

Two signals w and v are said to satisfy the IQC defined by Π if:

$$\int_{-\infty}^{\infty} \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} \hat{v}(j\omega) \\ \hat{w}(j\omega) \end{bmatrix} d\omega \geq 0 \qquad (1)$$

where $\hat{v}(j\omega)$ and $\hat{w}(j\omega)$ are the Fourier Transforms of signals w and v.
Consider the following system, illustrated in Figure 1:

$$\begin{aligned} v &= Gw + f \\ w &= \Delta(v) + e \end{aligned} \qquad (2)$$

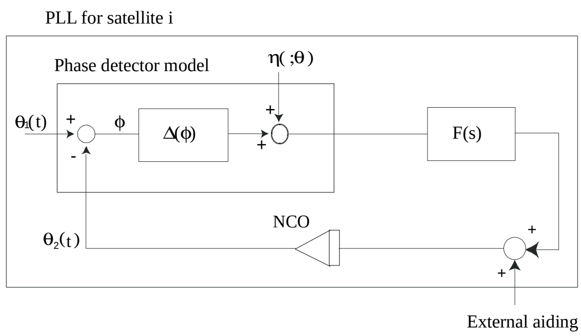
where G is a linear time-invariant operator with transfer function $G(s)$, and Δ represent the nonlinear, time-varying or uncertain component(s) of a system. Sufficient conditions for absolute stability of the feedback system (2) is:

1. There exist an $\varepsilon > 0$ such that:
$$\begin{bmatrix} G(j\omega) \\ I \end{bmatrix}^* \Pi(j\omega) \begin{bmatrix} G(j\omega) \\ I \end{bmatrix} \leq -\varepsilon I, \quad \forall \omega \in \mathbb{R} \quad (3)$$
2. The system is minimally stable.

The definition of minimal stability in general is rather technical, but for the sector nonlinearity we will consider in this paper, it is enough to verify that the feedback system is stable for all linear blocks $w = kv$ that satisfy (1). For a more detailed description of this theory, look in [3]. Note that if several IQCs with $\Pi_1, \cdots, \Pi_n$ can be found for Δ, then we can use $\Pi = \alpha_1\Pi_1 + \alpha_2\Pi_2 + \cdots + \alpha_n\Pi_n$ for any set of $\alpha_1, \alpha_2, \cdots, \alpha_n \geq 0$. This means that the more IQCs that can be found for Δ, the better.

3 GPS RECEIVER MODEL

GPS receiver operation can be divided into three modes



2. Carrier loop.

with increasing robustness to high levels of RF interference and dynamics. The first mode is normal operation, and the code loop is aided by the carrier loop. The code loop bandwith can be substantially reduced compared to the unaided case. As the level of interference and dynamics increases, the carrier loop will loose lock first. An INS LOS velocity signal can be applied to the carrier loop, while the carrier loop in its turn aids the code loop. This is the second mode. The third, and most robust mode, is to open the carrier loop so that INS aiding is provided directly to the code loop. We will first consider the first mode with an unaided third order carrier loop, and then the second mode, where a second order carrier loop is aided by an INS.

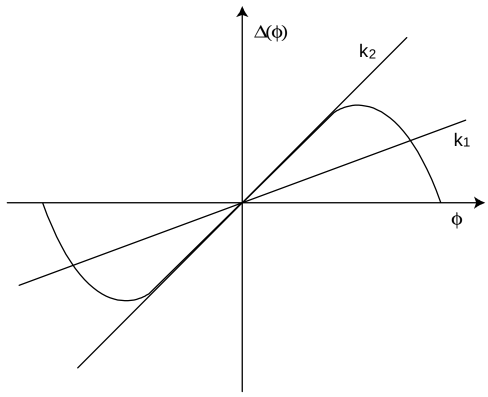
A simplified model of the carrier loop is shown in Figure 2. The function $\Delta(\phi)$ is the phase discriminator model which may be nonlinear. F(s) is the loop filter, which is assumed to be of second order when not aided, and of first order when aided. The total linear loop transfer function including the NCO, which is modeled as an integrator, is:

$$\text{Second order loop:} \quad G(s) = \frac{1}{s}F(s) = k\frac{(s+a)}{s^2} \tag{4}$$

$$\text{Third order loop:} \quad G(s) = \frac{1}{s}F(s) = k\frac{(s+a)(s+b)}{s^3} \tag{5}$$

4 IQC ANALYSIS OF A NONLINEAR CARRIER LOOP

The most common discriminator in modern digital receivers is the arctan function. It has a wide linear range at high carrier to noise (C/N_0) levels. However, as the C/N_0 decreases the discriminator characteristic tends towards a sinuoidal shape. It can be argued that for most



3. Sector nonlinearity.

discriminators, the charcteristics are nonlinear at low C/N_0 levels. The nonlinearity can be assumed to lie within a sector as illustrated in Figure 3. By inspecting the third order linear system, it is clear that instability is a result of low loop gain. The problem can then be stated as follows: To find the lowest possible slope k_1 that can guarantee a stable system for a given loop filter gain. The intersection of k_1 and the discriminator characteristic gives the largest phase error that can be tolerated with a given loop filter gain In terms of IQC stability this is refered to as absolute stability in a finite domain.

For a sector nonlinearity where we put $k_2 = 1$, several IQCs can be found:

$$(v-w)^* (w - k_1 v) + (w-v)^* (k_1 v - w) \geq 0 \tag{6}$$

$$w^*(w - k_1 v) + (w - k_1 v)^* w \geq 0 \tag{7}$$

The constraint (6) gives the following Π:

$$\Pi = \begin{bmatrix} -2k_1 & (1+k_1) \\ (1+k_1) & -2 \end{bmatrix} \tag{8}$$

Using (3) the following criterium is obtained:

$$-2k_1 G(-s)G(s) - (1+k_1)\left[G(s) + G(-s)\right] - 2 < 0$$

We obtain:

$$\begin{aligned} k_1 \quad &> \quad \frac{-(\operatorname{Re} G(j\omega) + 1)}{\operatorname{Im}^2 G(j\omega) + \operatorname{Re}^2 G(j\omega) + \operatorname{Re} G(j\omega)} \\ &= \quad \frac{k(a+b)\omega^4 - \omega^6}{(k^2 - k(a+b))\omega^4 + k^2(a^2+b^2)\omega^2 + k^2a^2b^2} \end{aligned}$$

This is actually a form of the well known circle criterion.

Example 1 *For a loop tuned to about 5 Hz noise bandwidth with optimum closed loop response, the sufficient* *condition for stability by this criterion is $k_1 > 0.34$. We now need to check for minimal stability. We can do this by replacing the nonlinear block with a linear gain, and use standard linear analysis to check for the minimum gain allowed. The result is:*

$$k_1 > \frac{ab}{(a+b)}\frac{1}{k} \tag{9}$$

which for this example results in $k_1 > 0.25$. Thus, absolute stability with a finite domain is achieved for $k_1 > 0.34$.

The constraint (7) gives the following Π:

$$\Pi = \begin{bmatrix} 0 & -k_1 \\ -k_1 & 2 \end{bmatrix} \tag{10}$$

This constraint is not useful, since the lower right corner of Π never should be positive. We can get around this by defining new variables, so that the problem is redefined into finding a maximum upper slope. The new variables are:

$$\begin{aligned} q &= \frac{1}{w}, \quad p = \frac{1}{v} \\ k_2 &= \frac{1}{k_1}, \quad H(s) = \frac{1}{G(s)} \end{aligned}$$

A useful constraint for this system is:

$$p^*(k_2 p - q(1+\eta j\omega)) + (k_2 p - q(1+\eta j\omega))^* p \geq 0$$

which gives

$$\Pi = \begin{bmatrix} 2k_2 & -(1+\eta j\omega) \\ -(1-\eta j\omega) & 0 \end{bmatrix}$$

Using (3), this leads to the well known Popov criterion:

$$\begin{aligned} 2k_2 H(j\omega)H(-j\omega) + (1-\eta j\omega)H(j\omega) &+ \\ (1+\eta j\omega)H(-j\omega) &< 0 \end{aligned} \tag{11}$$

This leads to:

$$\begin{aligned} & 2k_2 + [(1-\eta j\omega)G(-j\omega) + (1+\eta j\omega)G(j\omega)] \\ = \; & 2k_2 + 2k\left[-\frac{a+b}{\omega^2} - \frac{\eta\omega(ab-\omega^2)}{\omega^3}\right] < 0 \end{aligned}$$

Choosing $\eta = -\frac{a+b}{ab}$ gives:

$$k_2 < \frac{(a+b)k}{ab}$$

The criterion on the original gain k_1 is:

$$k_1 = \frac{1}{k_2} > \frac{ab}{(a+b)}\frac{1}{k}$$

This is identical to the linear stability criterion found in (9), and there is reason to believe that this is a very tight (close to necessary) criterion for stability. Thus, it is not necessary to look for more constraints. Note that with other transfer functions, we can in general *not* replace the nonlinearity with a linear block corresponding to the upper or lower bound of the nonlinearity, and analyze the system like a linear system.

5 IQC ANALYSIS OF A GPS RECEIVER AIDED BY AN INS

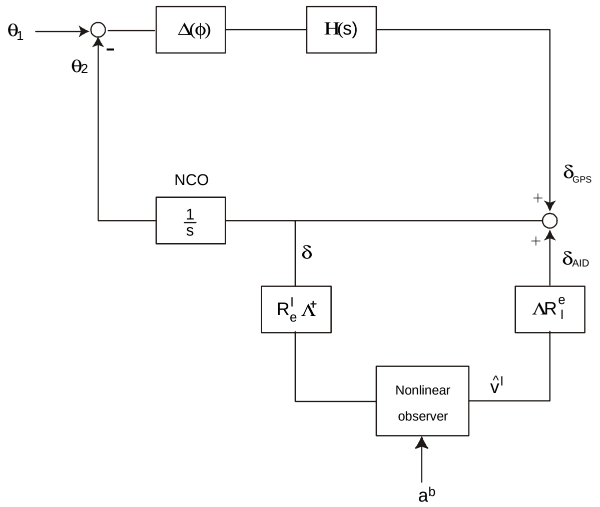
The purpose of this section is to see what result the IQC methodology can give when analyzing the total nonlinear feedback system. We will start by presenting the nonlinear observer from [4]:

$$\begin{aligned} \dot{\hat{\mathbf{v}}}^l &= \hat{\mathbf{R}}_b^l\left[(\mathbf{I}+\hat{\Delta})\mathbf{a}_{\text{imu}} + \hat{\mathbf{b}}_2\right] - \left[2\mathbf{S}(\omega_{ie}^l) + \mathbf{S}(\omega_{el}^l)\right]\hat{\mathbf{v}}^l \\ &\quad + \bar{\mathbf{g}}^l + \mathbf{K}_2\tilde{\mathbf{v}}^l \\ \dot{\hat{\mathbf{b}}} &= -\mathbf{T}_1^{-1}\hat{\mathbf{b}}_2 + \mathbf{K}_3(\hat{\mathbf{R}}_b^l)^T\tilde{\mathbf{v}}^l \\ \dot{\hat{\boldsymbol{\epsilon}}} &= -\mathbf{T}_2^{-1}\hat{\boldsymbol{\epsilon}} + \mathbf{K}_4\,\text{diag}(\mathbf{a}_{\text{imu}})(\hat{\mathbf{R}}_b^l)^T\tilde{\mathbf{v}}^l \\ \dot{\hat{\boldsymbol{\beta}}} &= -\mathbf{T}_3^{-1}\hat{\boldsymbol{\beta}} + \mathbf{K}_5\Upsilon^T(\mathbf{a}_{\text{imu}})(\hat{\mathbf{R}}_b^l)^{\mathbf{T}}\tilde{\mathbf{v}}^l \end{aligned} \tag{12}$$

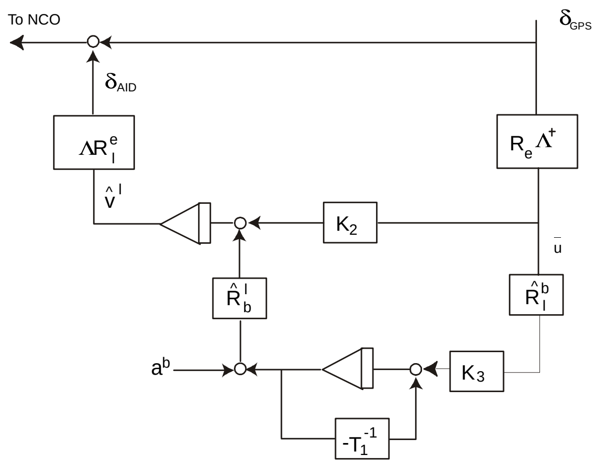
where Υ is a linear function, $\mathbf{S}$ is the cross product matrix, $\mathbf{K}_i = k_i\mathbf{I} > \mathbf{0}$, ($i = 2, 3, 4$) are three 3×3 gain matrices, and $\mathbf{K}_5 = k_5\mathbf{I}$ is a 6×6 gain matrix. $\hat{\Delta} = \hat{\Delta}(\boldsymbol{\epsilon}, \boldsymbol{\beta})$, $\boldsymbol{\epsilon} = [\epsilon_x, \epsilon_y, \epsilon_z]^T$ are estimates of three accelerometer scale factor errors, and $\hat{\boldsymbol{\beta}} = [\beta_{xy}, \beta_{xz}, \beta_{yx}, \beta_{yz}, \beta_{zx}, \beta_{zy}]^T$ are estimations of six accelerometer misalignment errors. $\hat{\mathbf{b}}$ represents the bias estimates of the accelerometers. $\mathbf{T}_i$ ($i = 1, 2, 3$) are large positive definite time constant matrices all assumed to be known. The filter update equations are:

$$\tilde{\mathbf{v}}^l = (\mathbf{R}_l^e)^T\Lambda^\dagger\left(\boldsymbol{\delta}_{\text{gps}} - \boldsymbol{\delta}_{\text{sat}} - \Lambda^e\mathbf{R}_l^e\hat{\mathbf{v}}^l\right) \tag{13}$$

where $\Lambda^e \in \Re^{nx3}$ is the LOS matrix for n satellites. The linear GPS error estimation equations and the position update normally included in the observer are not shown here. There are several nonlinearities in the above observer. The most dominant one is the rotation matrix $\hat{\mathbf{R}}_b^l$, that contains the attitude dynamics. In order to keep the complexity down, we will not consider the attitude part of the observer for this analysis, and only bias estimation will be considered. The rotation matrix $\mathbf{R}_e^l$ associated with the rotation of the ECEF frame relative to the local (NED) frame, and the LOS matrix Λ, will be assumed constant because of their relatively slow dynamics compared to $\hat{\mathbf{R}}_b^l$. The term $\left[2\mathbf{S}(\omega_{ie}^l) + \mathbf{S}(\omega_{el}^l)\right]\hat{\mathbf{v}}^l$ is small enough to be ignored in the stability analysis. The total system is illustrated in Figure 4.



4. The integrated GPS/INS system with the INS aiding the carrier tracking loop of the GPS receiver.



5. The figure shows the INS part of the integrated system when only bias estimation is included.

We now write the system on matrix form:

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x} + \mathbf{B}(t)\mathbf{u} \tag{14}$$

where

$$\mathbf{A}(t) = \begin{bmatrix} -\mathbf{K}_2 & \hat{\mathbf{R}}_b^l(t) \\ -\mathbf{K}_3\hat{\mathbf{R}}_l^b(t) & -\mathbf{T}_1^{-1} \end{bmatrix}, \mathbf{B}(t) = \begin{bmatrix} \mathbf{K}_2 \\ \mathbf{K}_3\hat{\mathbf{R}}_l^b(t) \end{bmatrix}$$

$$\mathbf{x} = \left[\left(\hat{\mathbf{v}}^l\right)^T, \left(\hat{\mathbf{b}}\right)^T \right]^T, \quad \mathbf{u} = (\mathbf{R}_l^e)^T \Lambda^\dagger \boldsymbol{\delta}$$

From Figure 4, we can see that $\boldsymbol{\delta} = \boldsymbol{\delta}_{\text{gps}} + \Lambda \mathbf{R}_l^e \hat{\mathbf{v}}^l$ which gives $\mathbf{u} = (\mathbf{R}_l^e)^T \Lambda^\dagger \boldsymbol{\delta}_{\text{gps}} + \hat{\mathbf{v}}^l = \bar{\mathbf{u}} + \hat{\mathbf{v}}^l$. This leads to the system:

$$\dot{\mathbf{x}} = \bar{\mathbf{A}}(t)\mathbf{x} + \mathbf{B}(t)\bar{\mathbf{u}}$$

where

$$\bar{\mathbf{A}}(t) = \begin{bmatrix} \mathbf{0} & \hat{\mathbf{R}}_b^l(t) \\ \mathbf{0} & -\mathbf{T}_1^{-1} \end{bmatrix}$$

The new system is illustrated in Figure 5.

An IQC for the observer can be found by defining $\mathbf{w}_2 = \hat{\mathbf{R}}_b^l(t)\hat{\mathbf{b}}$ and $\mathbf{v}_2 = \bar{\mathbf{u}}$. The following relation holds for all T:

$$\int_0^T \left(\mathbf{w}_2^T \hat{\mathbf{R}}_b^l(t) \mathbf{K}_3 \hat{\mathbf{R}}_l^b(t) \mathbf{v}_2 - \mathbf{w}_2^T \hat{\mathbf{R}}_b^l(t) \mathbf{T}_1^{-1} \hat{\mathbf{R}}_l^b(t) \mathbf{w}_2 \right) dt$$

$$= \int_0^T \left(\mathbf{w}_2^T \mathbf{K}_3 \mathbf{v}_2 - \mathbf{w}_2^T \mathbf{T}_1^{-1} \mathbf{w}_2 \right) dt \geq 0$$

This is the definition of a *strictly output passive* system. We can now set up the input and output relation for the two IQCs (see Figure 6):

$$\begin{aligned} v_1 &= -\mathbf{G}_1(s) w_1 - \mathbf{G}_2(s) w_2 \\ v_2 &= \mathbf{G}_3(s) w_1 \end{aligned}$$

where

$$\begin{aligned} \mathbf{G}_1(s) &= \frac{1}{s^3} (s\mathbf{I} + \Lambda \mathbf{K}_2 \Lambda^\dagger)(s\mathbf{I} + \mathbf{A}) \\ \mathbf{G}_2(s) &= \frac{1}{s^2} \Lambda \mathbf{R}_l^e \\ \mathbf{G}_3(s) &= \frac{1}{s} (\mathbf{R}_l^e)^T \Lambda^\dagger (s\mathbf{I} + \mathbf{A}) \end{aligned}$$

$\mathbf{K}_1 = k_1 \mathbf{I}$ and $\mathbf{A} = \text{diag}(a_i)$ where a_i is the zero of loop filter i. Looking at the total system without bias estimation, we can use the same Popov IQC for the discriminator nonlinearity as above with $\mathbf{G}_1(s)$ as the transfer function. Each of the carrier loops will function as a third order stand-alone loop, but they can now be tuned to a substantially lower bandwidth. The stability condition now is:

$$\mathbf{K}_1 - \mathbf{K}^{-1} \left[\mathbf{A} + \Lambda \mathbf{K}_2 \Lambda^\dagger \right]^{-1} \mathbf{A} \Lambda \mathbf{K}_2 \Lambda^\dagger > 0$$

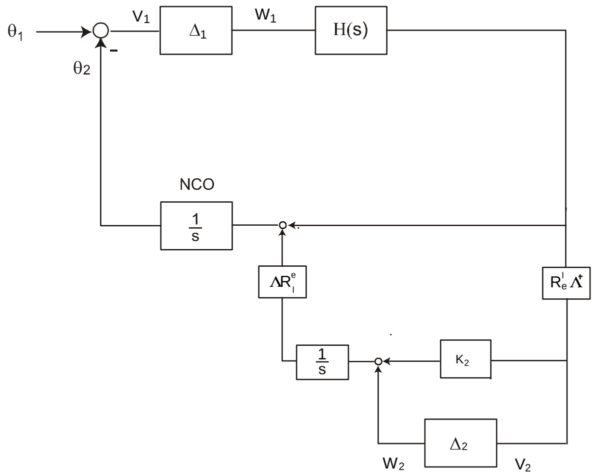
When we included bias estimation, the only IQC for the discriminator nonlinearity that gave a result was the circle criterion:

$$(\mathbf{v} - \mathbf{w})^* (\mathbf{w} - \mathbf{K}_1 \mathbf{v}) + (\mathbf{w} - \mathbf{v})^* (\mathbf{K}_1 \mathbf{v} - \mathbf{w}) \geq 0 \tag{15}$$

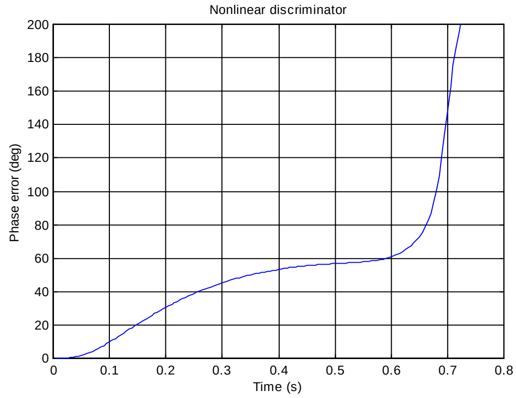
Π for the total system is:

$$\Pi = \begin{bmatrix} -2\mathbf{K}_1 & \mathbf{0} & \mathbf{K}_1 + \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \alpha \mathbf{K}_3 \\ \mathbf{K}_1 + \mathbf{I} & \mathbf{0} & -2\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \alpha \mathbf{K}_3 & \mathbf{0} & -2\alpha \mathbf{T}_1^{-1} \end{bmatrix}$$

$\alpha > 0$ is a constant weighing the two constraints. The con-



6. The figure shows the relations between IQC input and output signals.



7. The response of the loop with a sinusoidal phase discriminator. The loop gain is tuned according to linear analysis.

dition for stability can be found as follows:

$$\mathbf{G}^*\Pi\mathbf{G} < \mathbf{0}$$

where

$$\mathbf{G} = \begin{bmatrix} -\mathbf{G}_1 & -\mathbf{G}_2 \\ \mathbf{G}_3 & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$$

We get:

$$\begin{bmatrix} \mathbf{P} & \mathbf{S} \\ \mathbf{S}^T & \mathbf{R} \end{bmatrix} < 0 \qquad (16)$$

where:

$$\begin{aligned} \mathbf{P} &= -2\mathbf{K}_1\left[\mathbf{G}_1^T(-s)\mathbf{G}_1(s)\right] \\ &\quad -(\mathbf{K}_1+\mathbf{I})\left[\mathbf{G}_1^T(-s)+\mathbf{G}_1(s)\right]-2\mathbf{I} \\ \mathbf{S} &= -\left[\mathbf{G}_1^T(-s)+(\mathbf{K}_1{+}\mathbf{I})\right]2\mathbf{K}_1\mathbf{G}_2(s) \\ &\quad +\mathbf{G}_3^T(-s)\alpha\mathbf{K}_3 \\ \mathbf{R} &= -2\mathbf{K}_1\mathbf{G}_2^T(-s)_1\mathbf{G}_2(s)-2\alpha\mathbf{T}_1^{-1} \end{aligned}$$

In order to ensure that (16) is negative definite the following conditions must be fulfilled:

$$-2\mathbf{T}_1^{-1} < 0 \qquad (17)$$

$$\mathbf{P}-\mathbf{S}\mathbf{R}^{-1}\mathbf{S}^T > 0 \qquad (18)$$

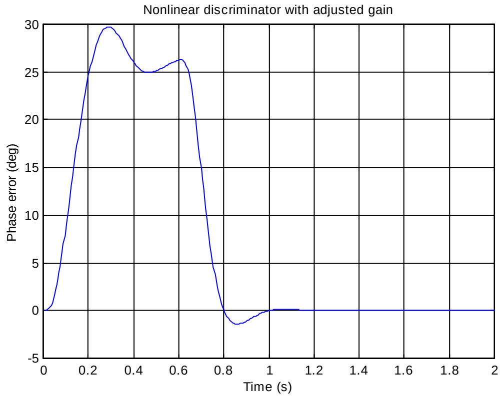
Evaluation of (18) is rather complex, and it gives a very conservative result.

6 SIMULATIONS

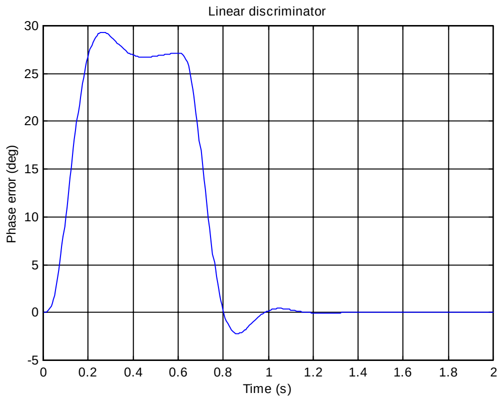
In these simulations, the carrier loop was tuned to 5 Hz, and a 10 g/s jerk stress was imposed for 0.6 seconds. This is a maximum jerk stress requirement used by several authors e.g. [1] ,and [5]. In order to isolate the effects of the nonlinear versus the linear dynamics of the loop, no noise was used in the simulations. The nonlinear phase discriminator used in the simulations were $0.5\sin(2\phi)$, which approximate the characteristic of a two-quadrant arctan discriminator under low C/N_0 conditions. The linear discriminator used is the two quadrant arctan function. Figure 7 confirmes that the system is unstable with the nonlinear discriminator when tuned to optimal performance using linear analysis. The same simulation, but now with a perfect linear discriminator, is shown in Figure 9. In Figure 8, the gain is increased to give approximately the same performance with the nonlinear discriminator as with the linear discriminator. The result support the statement that the stability condition probably is both suffcient and necessary.

7 CONCLUSIONS AND FUTURE WORK

In this paper we have presented the concept of Integral Quadratic Constraints, which has proven useful when analyzing systems with both linear and nonlinear blocks. A sufficient condition for stability of third order tracking loops with a sector nonlinearity has been derived. Simulations indicated that the condition probably is necessary as well as sufficient. When applied to the more complex system of an integrated GPS/INS system, the methodology has so far given a very conservative result when bias estimation is included. However, when error estimation is disregarded, the resulting linear transfer function of a second order tracking loop combined with INS velocity dynamics is equal to that of a third order tracking loop. The same analysis can therefore be applied. Future work will include further investigation of other IQCs as well



8. The response of the loop with a sinusoidal phase discriminator. The loop gain is tuned according to nonlinear analysis.



9. The response of the loop with a linear phase discriminator. The loop gain is tuned according to linear analysis.

as attempts to utilize other nonlinear analysis techniques to give better analytic conditions for stability of the integrated GPS/INS system. Moreover, nonlinear stochastic theory for tracking loops will be looked into.

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