A Model Based Wave Filter for Surface Vessels Using Position, Velocity and Partial Acceleration Feedback

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Abstract

We propose a scalable state estimator with wave filter for surface vessels. The filter is shown to be either globally or locally exponentially stable depending on configuration. Performance is illustrated using actual data from a full scale semi-submersible drilling rig.

1 Introduction

Classic solutions to the dynamic positioning (DP) problem of surface vessels are output-feedback designs using a state-estimator to filter out 1st-order wave motion from the low-frequency (LF) positions while reconstructing LF velocities [1, 2, 3, 4, 5, 6]. All these were realized using linear stochastic theory (Kalman Filter), but also H_∞-solutions been proposed [7]. Unfortunately, the linearization of the nonlinear kinematics implies that the results are only valid locally. However, if the nonlinearities satisfy a global Lipschitz-condition, a modification [8] of the extended Kalman filter ensures global exponential stability. Another approach with comparable performance is to utilize the model structure and let the observer "linearize" itself around the measured compass heading. As opposed to traditional extended Kalman-filters, the on-line explicit linearization is avoided, and global stability properties are more easily established since the nonlinear kinematics can be treated as a known time-varying block. Examples are the passivation designs of Fossen and Strand [9, 10], extensions to higher order monotonic damping terms [11] and non-dissipative linear damping terms [12].

Common for all of these filters is that they are derived under the assumption that only the positions and compass heading were available for feedback. Today high performance inertial measurement units (IMU) are becoming increasingly affordable and integrated navigation systems (INS) that integrate IMU and GPS reproduce not only positions but also velocities and linear accelerations with great accuracy to a reasonable price. This development in sensor technology calls for an update of the existing filters/estimators being used in commercial DP systems. Two important aspects to consider are the incorporation of the "new" measurements available and to seek solutions that are valid globally in the sense that linearizations of the kinematics should be avoided.

Using the measured accelerations, we are able to better keep up with unmodeled disturbances like slowly varying wave forces which must be counteracted by the control system. Slowly varying wave induced forces is a phenomenon well known from nonlinear hydrodynamic theory [13], yet they are difficult to express in a form suited for control design. However, feeding the measured accelerations uncritically into the closed loop system is not recommended due to the high-frequency, large amplitude oscillations caused by 1st-order wave loads. Therefore, some kind of notch filtering of the measured accelerations is required in order to remove the wave frequency components.

Referring to the above, there already exists a variety of state-estimators with wave filtering capabilities that utilizes position measurements to reconstruct LF positions and velocities. Rather than using separate filters for the acceleration and velocity signals, it seems more attractive to design a combined filter/estimator system considering all available measurement data together.

The main motivating factor for using a model based filter is *dead-reckoning*, that is if the sensors should fail, the observer will for a period of time still allow a reasonable accurate estimation of the actual states based on the control input alone. In fact, this is a system redundancy requirement which is mandatory with the classification societies [14].

The idea behind the filter proposed is: An existing model based observer [9, 12] has been extended to optionally include velocity and/or linear acceleration measurements. By assuming that the yaw rate is bounded, we show that the filter error dynamics is uniformly locally exponentially stable (ULES), a very attractive property because it allows the observer and controller to be independently tuned (nonlinear separation principle) without possible closed-loop instability [15].

In Section 2 below we describe the vessel model and some of its properties with attention given to how the individual measurements fit into the model framework. Section 3 concerns the filter design and stability analysis. An important part of this section is the discussion regarding structural properties: The structure of gain matrices updating the Earth-fixed error dynamics can not be selected arbitrarily. Section 4 is dedicated to illustrating the observer's perfor-

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mance using recorded data from a semi-submersible drilling rig. Section 5 presents our conclusions.

2 Problem Statement

2.1 Vessel and Environment Model

We choose to express the model in the Earth- and body-fixed coordinate frames. The body-fixed frame coincides with the principal axes of the vessel and it is rotated an angle ψ_y wrt. the Earth-fixed frame. This transformation of coordinates is represented by the orthogonal rotation matrix:

$$R(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Let $x_w \in \mathbb{R}^{3n_w}$ describe the first order wave-induced motion where n_w denotes the number of states used to describe the wave frequency motion in each degree of freedom (DOF). $b \in \mathbb{R}^3$ is an unknown bias force due to wind, current and higher order wave loads, $\eta = [x, y, \psi]^T$ is the LF position where x and y is the North and East positions respectively, and ψ being the LF heading. $\nu = [u, v, r]^T$ is the LF body-fixed velocities, i.e. surge, sway and yaw. The system model is assumed to satisfy:

A1 The orientation angle between the Earth-fixed and body-fixed frame is the measured heading ψ_y such that:

$$\begin{aligned} \dot{x}_w &= A_w x_w + E_w w_w && (2) \\ \dot{\eta} &= R(\psi_y)\,\nu && (3) \\ \dot{b} &= -T_b^{-1} b + E_b w_b && (4) \\ M\dot{\nu} &= -GR^T(\psi_y)\eta - D\nu + \tau + R^T(\psi_y)b && (5) \end{aligned}$$

where $\tau \in \mathbb{R}^3$ is the applied thruster force, $M = M^T > 0$ is the mass, $D \in \mathbb{R}^{3\times3}$ contains linear damping coefficients and $G \in \mathbb{R}^{3\times3}$ describes the mooring forces. The bias forces b are modelled as Markov processes with a diagonal matrix $T_b \in \mathbb{R}^{3\times3}$ of time constants. $A_w \in \mathbb{R}^{3n_w\times3n_w}$ is assumed Hurwitz and describes the first order wave induced motion (see Section 2.2). The wave and bias models are driven by Gaussian disturbances $w_w \in \mathbb{R}^3$ and $w_b \in \mathbb{R}^3$, respectively. See [9] for a more detailed model description.

In the following we will frequently utilize a commutation property between the Earth-fixed parameters and the rotation $R(\alpha)$.

Property 1 *A matrix $A \in \mathbb{R}^{3\times3}$ is said to* commute *with the rotation $R(\alpha)$ if*

$$AR(\alpha) = R(\alpha)A \quad (6)$$

Examples of matrices A satisfying Property 1 are linear combinations $A = a_1R(\theta) + a_2I + a_3k^Tk$ for scalars a_i, θ and $k = [0, 0, 1]^T$, the axis of rotation. Also note that since $R(\alpha)$ is orthogonal, that is $R^T(\alpha) = R^{-1}(\alpha)$, Property 1 implies that

$$A = R^T(\alpha)AR(\alpha) = R(\alpha)AR^T(\alpha) \quad (7)$$

The model assumptions are:

A2 For $D = \{d_{ij}\}$ the elements $d_{11}, d_{22} > 0$.

A3 The bias time constant matrix T_b and each 3×3 sub-block of A_w satisfies Property 1.

Note in assumption A2 that there are no restrictions on neither d_{23}, d_{32} nor d_{33}. A2 is thus less restrictive than assuming $D + D^T > 0$ [9] and its interpretation is that separate surge and sway motions are dissipative. The last assumption, A3, implies that the mean wave motion period, relative damping and bias time constants in the North and East directions are identical. It should be emphasized that this is not as restrictive as it may sound since the dominating frequency of the first order wave induced motions will be approximately the same in surge and sway.

Collect the Earth-fixed states in $x_1 \in \mathbb{R}^{6+3n_w}$ and the body-fixed in $x_2 \in \mathbb{R}^3$

$$\begin{aligned} x_1 &= \begin{bmatrix} x_w^T & \eta^T & b^T \end{bmatrix}^T && (8) \\ x_2 &= \nu && (9) \end{aligned}$$

and define the block diagonal transformation matrix

$$T(\psi_y) = \mathrm{diag}(R^T(\psi_y), .., R^T(\psi_y), I) \quad (10)$$

On compact form using Assumption A3 we get with $x = [x_1^T, x_2^T]^T$ and $w = [w_w^T, w_b^T]^T$

$$\dot{x} = T^T(\psi_y)AT(\psi_y)x + B\tau + Ew \quad (11)$$

where

$$A = \begin{bmatrix} A_w & 0 & 0 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & -T_b^{-1} & 0 \\ 0 & -M^{-1}G & M^{-1} & -M^{-1}D \end{bmatrix} \quad (12)$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ M^{-1} \end{bmatrix}, \quad E = \begin{bmatrix} E_w & 0 \\ 0 & 0 \\ 0 & E_b \\ 0 & 0 \end{bmatrix} \quad (13)$$

2.2 First Order Wave Motion

We assume that the power spectrum of the oscillatoric motion caused by the 1st-order wave loads can be approximated by a set of three decoupled linear transfer functions

$$h(s) = \frac{k_{wi}s}{(s^2 + 2\zeta_i\omega_{0i}s + \omega_{0i}^2)^2} \quad (14)$$

in each of the 3 DOF. The motivation for using a function of fourth order instead of other approximations, is that this choice ensures that the transfer functions between the excitation and positions, velocities as well as accelerations, will be strictly proper.

A minimal realization with $x_w \in \mathbb{R}^{12}$ can in state-space be described by

$$\dot{x}_w = A_w x_w + E_w w_w \tag{15}$$

$$A_w = \begin{bmatrix} 0 & I & 0 & 0 \\ -\Omega & -\Lambda & 0 & I \\ 0 & 0 & 0 & I \\ 0 & 0 & -\Omega & -\Lambda \end{bmatrix}, \quad E_w = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \operatorname{diag}(e_i) \end{bmatrix}$$

Ω and Λ are diagonal matrices holding the the wave motion resonance frequencies ω_{oi} and relative damping factors ζ_i, $i = 1..3$ for the North, East and heading respectively like this

$$\begin{aligned} \Omega &= \operatorname{diag}(\omega_{01}^2, \omega_{02}^2, \omega_{03}^2) && (16) \\ \Lambda &= \operatorname{diag}(2\zeta_1\omega_{01}, 2\zeta_2\omega_{02}, 2\zeta_3\omega_{03}) && (17) \end{aligned}$$

Assumption A3 requires $\omega_{01} = \omega_{02}$ and $\zeta_1 = \zeta_2$. The Earth-fixed wave induced position, velocity and acceleration can be extracted from x_w as follows

$$p_w = C_p x_w, \quad v_w = C_v x_w, \quad a_w = C_a x_w \tag{18}$$

where $C_p, C_v, C_a \in \mathbb{R}^{3\times 12}$

$$\begin{aligned} C_p &= \begin{bmatrix} I & 0 & 0 & 0 \end{bmatrix} && (19) \\ C_v &= \begin{bmatrix} 0 & I & 0 & 0 \end{bmatrix} && (20) \\ C_a &= \begin{bmatrix} -\Omega & -\Lambda & 0 & I \end{bmatrix} && (21) \end{aligned}$$

Since $\frac{d}{dt}\left(R^T v_w\right) = \frac{d}{dt}\left(R^T\right) v_w + R^T \dot{v}_w = -\dot{\psi}_y R^T S v_w + R^T a_w$, we get that in the body-fixed frame the experienced wave induced velocities and accelerations are

$$\begin{aligned} v_w^B &= R^T(\psi_y) v_w = R^T(\psi_y) C_v x_w && (22) \\ a_w^B &= R^T(\psi_y)\left(C_a - \dot{\psi}_y S C_v\right) x_w && (23) \end{aligned}$$

with

$$S = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{24}$$

The acceleration term depending on measured rotation rate $\dot{\psi}_y$ can be regarded as a Coriolis-like term.

2.3 Measurements

We intend to cover all combinations of position, velocity and acceleration measurements. The positions are usually given in an Earth-fixed reference frame, while velocities and accelerations are given in a body-fixed coordinate system.

There might be cases where not all kinds of measurements are available. Either due to sensor failure, or simply because that particular vessel was not equipped with that kind of instrument. Denote the measurements y. We have that $y \in \mathbb{R}^{n_y}$ where $3 \le n_y \le 8$ depending on the configuration. Define Υ_2 and Υ_3 as the projections extracting the measured velocities and accelerations respectively from the actual three DOF velocity and accelerations vectors. It is possible to measure all three velocities pretty accurately, which means that quite often $\Upsilon_2 = I$. While linear accelerations are easy the measure, the angular acceleration is not. Therefore, most likely only the accelerations in surge and sway are available and hence $n_{y_3} = 2$ and

$$\Upsilon_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{25}$$

Let $y_1 \in \mathbb{R}^3$ contain the earth-fixed positions and compass heading, $y_2 \in \mathbb{R}^{n_{y2}}$ the vessel-fixed velocities and $y_3 \in \mathbb{R}^{n_{y3}}$ accelerations. Then,

$$\begin{aligned} y_1 &= \eta + \eta_w = \eta + C_p x_w && (26) \\ y_2 &= \Upsilon_2\left(\nu + R^T C_v x_w\right) && (27) \\ y_3 &= \Upsilon_3\left(\dot{\nu} + a_w^V\right) \\ &= \Upsilon_3 M^{-1}\left(-G R^T \eta + R^T b - D\nu\right) \\ &\quad + \Upsilon_3 R^T\left(C_a - \dot{\psi}_y S C_v\right) x_w && (28) \end{aligned}$$

Compactly written

$$y = C_y(\psi_y, \dot{\psi}_y)x + D_y \tau \tag{29}$$

where

$$C_y(\psi_y, \dot{\psi}_y) = \begin{bmatrix} C_p & I & 0 & 0 \\ \Upsilon_2 C_v R^T & 0 & 0 & \Upsilon_2 \\ \Upsilon_3(C_a - \dot{\psi}_y S C_v)R^T & -\Upsilon_3 M^{-1} G R^T & \Upsilon_3 M^{-1} R^T & -\Upsilon_3 M^{-1} D \end{bmatrix}$$

$$D_y = \begin{bmatrix} 0 & 0 & M^{-T}\Upsilon_3^T \end{bmatrix}^T \tag{30}$$

When only positions are available, $n_{y_2} = n_{y_3} = 0$, the model is reduced the traditional DP observer problem.

It will be convenient in the stability analysis below to make an assumption on how the velocity and acceleration feedback is configured:

A4 Let $\Pi_i = \Upsilon_i^T \Upsilon_i \in \mathbb{R}^{3\times 3}$, $i = 2, 3$. Valid configurations are those that allow Π_i to commute with $R(\alpha)$, that is $R(\alpha)\Pi_i = \Pi_i R(\alpha)$.

2.4 Objective

For the model (11) with output (29)-(30), under Assumptions A1–A3, we seek a deterministic, model based observer that is exponentially stable for all possible sensor combinations satisfying A4.

3 Observer

By duplicating the system dynamics (11), the following observer is proposed:

$$\dot{\hat{x}} = T^T(\psi_y) A T(\psi_y)\hat{x} + K(\psi_y)\tilde{y} \tag{31}$$

The estimated output is

$$\hat{y} = C_y(\psi_y, \dot{\psi}_y)\hat{x} + D_y \tau \tag{32}$$

and hence when the estimation error is $\tilde{x} = x - \hat{x}$,

$$\tilde{y} = y - \hat{y} = C_y(\psi_y, \dot{\psi}_y)\tilde{x} \tag{33}$$

Although it somewhat restricts the flexibility, we suggest not to update the Earth-fixed estimates from the acceleration error $\tilde{y}_3$ at this stage. Therefore, this particular observer gain matrix $K(\psi_y)$ with constant $K_{1i} \in \mathbb{R}^{12\times 3}$, $K_{2i} \in \mathbb{R}^{3\times 3}$, $K_{3i} \in \mathbb{R}^{3\times 3}$ and $K_{4i} \in \mathbb{R}^{3\times 3}$ is suggested

$$K(\psi_y) = \begin{bmatrix} K_{11} & K_{12}R\Upsilon_2^T & 0 \\ K_{21} & K_{22}R\Upsilon_2^T & 0 \\ K_{31} & K_{32}R\Upsilon_2^T & 0 \\ K_{41}R^T & K_{42}\Upsilon_2^T & K_{43}\Upsilon_3^T \end{bmatrix} \tag{34}$$

where the following assumption is made:

A5 Each 3×3 sub-block of the gain matrices K_{ji}, $1 \leq j \leq 3$, $i = 1, 2$ commute with $R(\alpha)$.

This implies that the gains in North and East must be identical. The body-fixed gain matrices K_{4i}, $1 \leq i \leq 3$ can, however, be selected freely.

3.1 Error Dynamics

Since ψ_y and the constant parameter matrix A are assumed known, we obtain:

$$\dot{\tilde{x}} = T^T(\psi_y)AT(\psi_y)\tilde{x} - K(\psi_y)C_y(\psi_y, \dot{\psi}_y)\tilde{x} \tag{35}$$

which can be written:

$$\dot{\tilde{x}} = T^T(\psi_y)A_o(\dot{\psi}_y)T(\psi_y)\tilde{x} \tag{36}$$

Assumptions A3–A5 are sufficient requirements for this. Moreover, it can be shown that the resulting A_o can be written as

$$A_o(\dot{\psi}_y) = A_0 + \dot{\psi}_y A_1 \tag{37}$$

$$A_0 = \begin{bmatrix} A_{0,11} & A_{0,12} \\ A_{0,21} & A_{0,22} \end{bmatrix} \qquad A_1 = \begin{bmatrix} 0 & 0 \\ A_{1,21} & 0 \end{bmatrix} \tag{38}$$

where $A_{0,11} \in \mathbb{R}^{18\times 18}$, $A_{0,12} \in \mathbb{R}^{18\times 3}$, $A_{0,21} \in \mathbb{R}^{3\times 18}$, $A_{0,22} \in \mathbb{R}^{3\times 3}$ and $A_{1,21} \in \mathbb{R}^{3\times 18}$. Denote $\bar{K}_{43} = I - K_{43}\Pi_3$ such that $K_{43}\Pi_3 = I - \bar{K}_{43}$. Then:

$$A_{0,11} = \begin{bmatrix} A_w - K_{11}C_p - K_{12}\Pi_2 C_v & -K_{11} & 0 \\ -K_{21}C_p - K_{22}\Pi_2 C_v & -K_{21} & 0 \\ -K_{31}C_p - K_{32}\Pi_2 C_v & -K_{31} & -T_b^{-1} \end{bmatrix} \tag{39}$$

$$A_{0,12} = \begin{bmatrix} -K_{12}\Pi_2 \\ I - K_{22}\Pi_2 \\ -K_{32}\Pi_2 \end{bmatrix} \tag{40}$$

$$A_{0,21} = \begin{bmatrix} -\left(K_{41}C_p + K_{42}\Pi_2 C_v + \left(I - \bar{K}_{43}\right)C_a\right)^T \\ -\left(K_{41} + \bar{K}_{43}M^{-1}G\right)^T \\ -\left(\bar{K}_{43}M^{-1}\right)^T \end{bmatrix}^T \tag{41}$$

$$A_{0,22} = \left[-K_{42}\Pi_2 - \bar{K}_{43}M^{-1}D\right] \tag{42}$$

$$A_{1,21} = \left(I - \bar{K}_{43}\right)SC_v \tag{43}$$

3.2 Stability Analysis

The form of the observer error dynamics is very attractive because the known transformation $T(\psi_y)$ can be eliminated from the analysis when Assumptions A3-A5 are employed. Although the eigenvalues of $T^T A_o(\dot{\psi}_y)T$ are identical to the ones of $A_o(\dot{\psi}_y)$ since $T^T(s) = T^{-1}(s)$ for all s, $\mathrm{Re}(\lambda_i(T^T A_o T)) < 0$ if and only if A_o is Hurwitz. In general an eigenvalue analysis of a linear time-varying system will not be sufficient to prove stability [16]: We have to find a Lyapunov function candidate to conclude on that.

The idea is to analyze the error-dynamics in the vessel-fixed coordinate system and selecting a quadratic Lyapunov function candidate $V = z^T P z$ where the P-matrix also satisfies some structural constraints. The following lemma will be useful in that respect [12].

Lemma 2 *Linear time-varying systems on the form*

$$\dot{\xi}_1 = \bar{A}_{11}\xi_1 + H(\phi)\bar{A}_{12}\xi_2 \tag{44a}$$

$$\dot{\xi}_2 = \bar{A}_{21}H^T(\phi)\xi_1 + \bar{A}_{22}\xi_2 \tag{44b}$$

$\xi_1 \in \mathbb{R}^{n_1}$, $\xi_2 \in \mathbb{R}^{n_2}$ *interconnected by a rotation* $H : \mathbb{R} \to \mathbb{R}^{n_1\times n_1}$ *and where* $\phi : \mathbb{R}_{\geq 0} \to \mathbb{R}$ *is a known signal and*

$$\bar{A}_{11} = H\bar{A}_{11}H^T \tag{45}$$

are uniformly globally exponentially stable (UGES) if there for a $Q = Q^T > 0$ *exists a structurally constrained* $P = P^T > 0$

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}, \quad \begin{aligned} P_{11}\dot{H}^T H &= -H^T\dot{H}P_{11} \\ H^T\dot{H}P_{12} &= 0 \end{aligned} \tag{46}$$

such that

$$P\bar{A} + \bar{A}^T P \leq -Q \tag{47}$$

where $\bar{A}$ *is the system matrix*

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \tag{48}$$

The system matrix $\bar{A}$ must be Hurwitz, otherwise no such P can be found.

Proof: Define $\xi = [\xi_1^T, \xi_2^T]^T$. Because of the structural constrains (45) on $\bar{A}_{11}$ we may write

$$\dot{\xi} = T^T(\phi)\bar{A}T(\phi)\xi \tag{49}$$

where

$$T(\phi) = \begin{bmatrix} H^T(\phi) & 0 \\ 0 & I \end{bmatrix} \tag{50}$$

Define $z = T(\phi)\xi$. Since $H^T(\phi)$ is a rotation, $\|z\| = \|\xi\|$. Now, abusing the notation slightly, $\dot{T}(\phi) = \frac{d}{dt}(T(\phi))$, we get

$$\begin{aligned} \dot{z} &= \dot{T}(\phi)\xi + T(\phi)\dot{\xi} = \left(\dot{T}(\phi) + \bar{A}T(\phi)\right)\xi \\ &= \left(\dot{T}(\phi)T^T(\phi) + \bar{A}\right)z \end{aligned} \tag{51}$$

such that differentiating $V = z^T P z$ along the trajectories yields

$$\begin{aligned} \dot{V} &= z^T P \dot{z} + z^T P \dot{z} \\ &= z^T \left(P\bar{A} + \bar{A}^T P \right) z + z^T \left(P\dot{T}T^T + T\dot{T}^T P \right) z \\ &\leq -z^T Q z + z^T \left(P\dot{T}T^T + T\dot{T}^T P \right) z \end{aligned} \qquad (52)$$

The structural constraints on P imply that the last term is zero and thus UGES is proven. ∎

Even though this lemma indeed provides sufficient conditions for the elimination of the kinematics term and the dependence on the varying signal ψ_y, we still have to deal with the varying parameter $\dot{\psi}_y$. Physically $\dot{\psi}_y$ describes the yaw rate of the vessel and intuitively this quantity will be quite limited even when exposed to incoming waves. Therefore, if a set of simultaneous Lyapunov inequalities are satisfied at the minimum and maximum of $\dot{\psi}_y$, the error dynamics (36) will be ULES. This is our main result and is summarized in the following theorem.

Theorem 3 *Consider the observer (31)-(34) and let the Earth-fixed observer gains be selected according to Assumption A5. Assume that:*

$$A_o(\delta) = A_0 + \delta A_1 \qquad (53)$$

is Hurwitz at $\delta = 0$. If there exists a $P = P^T > 0$

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \qquad (54)$$

where there are structural constraints on P_{11} and P_{12}

$$\begin{aligned} P_{11}\dot{H}^T H &= -H^T \dot{H} P_{11} & (55) \\ H^T \dot{H} P_{12} &= 0 & (56) \end{aligned}$$

and an $\varepsilon > 0$ such that for $\underline{\delta} = \min_t \dot{\psi}_y$ and $\bar{\delta} = \max_t \dot{\psi}_y$ the simultaneous Lyapunov inequalities are satisfied

$$\begin{aligned} PA_o(\underline{\delta}) + A_o^T(\underline{\delta})P &\leq -\varepsilon I & (57) \\ PA_o(\bar{\delta}) + A_o^T(\bar{\delta})P &\leq -\varepsilon I & (58) \end{aligned}$$

the error-dynamics (36) is uniformly semi-globally exponentially stable (ULES).

Proof: $A_o(0) = A_0$ being Hurwitz is an obvious requirement, likewise is the Lyapunov inequalities (57)-(58) sufficient for ensuring that for all $\delta \in [\underline{\delta}, \bar{\delta}] \subset \mathbb{R}$ since

$$f(\delta) = x^T \left(PA_o(\delta) + A_o^T(\delta)P \right) x \qquad (59)$$

is linear in δ and thus convex such that if $f(\underline{\delta}), f(\bar{\delta}) \leq -\varepsilon$, $f(\delta) \leq -\varepsilon$ for any $\delta \in [\underline{\delta}, \bar{\delta}]$.

The rest of the proof consists of verifying that the error-dynamics can be expressed as (44) such that Lemma 2 can be employed. ∎

Remark 4 *For configurations where only position and/or velocity feedback are used, $A_1 = 0$ and the assumption of $\dot{\psi}_y$ being bounded is removed. The problem is thus reduced to finding a suitable $P = P^T > 0$ such that $PA_0 + A_0^T P \leq -\varepsilon I$. In those cases, the observer is UGES.*

Remark 5 *This approach to handling the varying $\dot{\psi}_y$ is conservative in the sense that guarantees exponential stability for arbitrarily fast variations in $\dot{\psi}_y$, i.e. as long as $\dot{\psi}_y$ is bounded there is no bound on $|\ddot{\psi}_y|$.*

The simultaneous Lyapunov inequalities can be represented as an LMI feasibility problem and hence solved using standard software packages: Find a $P = P^T > 0$ in accordance with the structural requirements such that

$$\begin{bmatrix} PA_o(\underline{\delta}) + A_o^T(\underline{\delta})P & 0 \\ 0 & PA_o(\bar{\delta}) + A_o^T(\bar{\delta})P \end{bmatrix} < 0 \qquad (60)$$

4 Full-Scale Experiments

Actual recordings of positions, compass heading and body-fixed velocities of a full-scale semi-submersible drilling rig is used to demonstrate the performance. The vessel was exposed to varying winds $10.5 - 14.0$ m/s ($20 - 27$ knots) and the dominating wave period was about $T_p = 11$ seconds with a significant wave height of 3.5 meters. Earth-fixed positions y_1 and body-fixed velocities y_2 where measured.

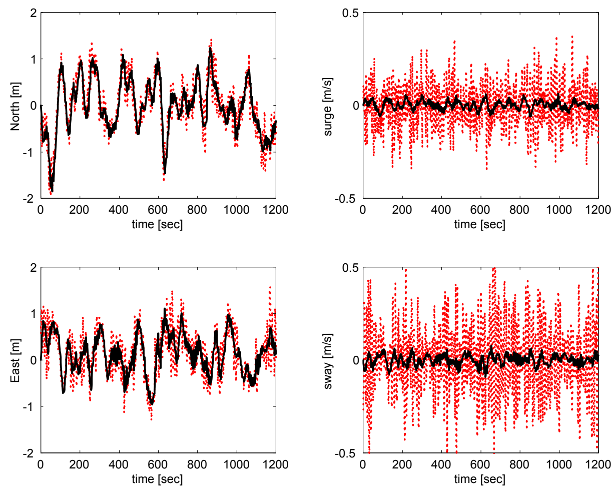

Figure 1: Left: Measured (dotted) positions and estimated LF-positions (solid). Right: Measured velocities (dotted) and estimated LF-velocities (solid).

The observer gains K_{ij} are all constant matrices and because the rotation $R(\psi_y)$ does not have any influence on the stability properties, tuning the filter can be done using linear techniques such as pole-placement similar to the algorithm proposed in [9, 15]. Another way is to perform an off-line curve-fit to minimize the squared error of the innovation $\tilde{y}$

$$\sigma_{\min} = \min_K \sum_{j=1}^{N} \left(y(t_j) - \hat{y}(t_j) \right)^2 \qquad (61)$$

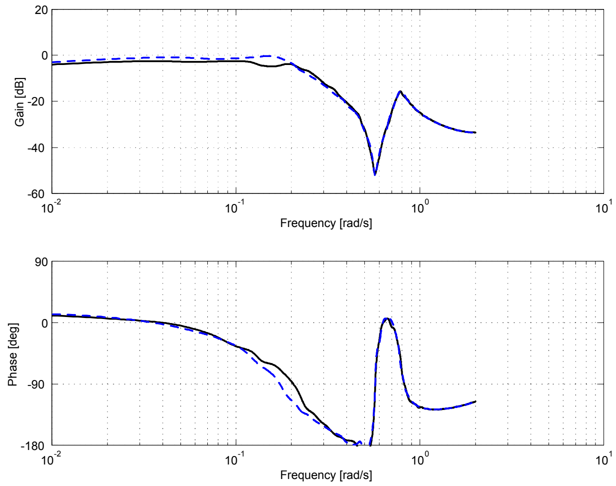

Figure 2: Calculated transfer function between measured velocity y_2 and LF-velocity $\hat{\nu}$ in surge (solid) and sway (dashed).

The latter approach was successfully pursued here. Figure 1 shows that the measured positions y_1 and velocities y_2 are corrupted with a fast oscillating signal due to the first order wave loads and that the low-frequency estimates $\hat{\eta}$ and $\hat{\nu}$ are left more or less unaffected by those rapid variations. Note however the large and slow variations, period $T > 120$ sec., resulting from higher order nonlinear wave effects, and the oscillations with period $T \approx 40$ sec., which equals the resonance frequency in the coupled surge and pitch motions. The calculated transfer functions between measured velocity y_2 and estimated LF-velocity $\hat{\nu}$ (Figure 2) verify that frequency components around $\omega = 2\pi/T_p \approx 0.57$ are attenuated. The same pattern could be seen between y_1 and $\hat{\eta}$.

5 Concluding Remarks

An existing nonlinear model-based observer with wave filtering capabilities for surface vessels has been extended to optionally include velocity and acceleration measurements. If the environmental model and some of the gain matrices satisfy certain structural properties, exponential stability of the filter error dynamics can be concluded using a quadratic Lyapunov function with structural constraints.

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