Formation Control of Marine Surface Craft using Lagrange Multipliers

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***Abstract*— We propose a method for constructing control laws for formation control of marine craft using classical tools from analytical mechanics. The control law is based on applying inter-vessel constraint functions which again impose forces on the individual vessels that maintain the given constraints on the total system. In this way, a formation can be assembled and stay together when exposed to external forces. A brief comparison with other control designs for a group of marine craft is done. Further, control laws for formation assembling (with dynamic positioning), and formation keeping during maneuvering is derived and simulated to illustrate the properties of the proposed method.**

I. Introduction

The fields of coordination and formation control with applications towards mechanical systems, ships, aircraft, unmanned vehicles, spacecraft, etc., have been the object of recent research efforts in the last few years. This interest has gained momentum due to technological advances in the development of powerful control techniques for single vehicles, the increasing computation and communication capabilities, and the ability to create small, low-power and low-cost systems. But coordinated control of several systems also means control of a more complicated system with challenges such as environmental information, communication between systems, etc [26].

Researchers have also been motivated by formation behaviors in nature, such as flocking and schooling, which benefits the animals in different ways [20], [22], [21], [14]. Many models from biology have been developed to give an understanding of the traffic rules that govern fish schools, bird flocks, and other animal groups, which again have provided motivation for control synthesis and computer graphics, see [24], [25] and the references therein. The interest in cooperative behavior in biological systems, and the mixture with the control systems field, have led to observations and models that suggest that the motion of groups in nature applies a distributed control scheme where the members of the formation are constrained by the position, orientation, and speed of their neighbors [20].

In this paper, we focus on the formation control aspect in cooperative behavior of marine craft. There exists a large number of publications on the fields of cooperative and formation control – recent results can be found in [14], [6], [30], [4] and [19]. See the papers and references therein for a thorough overview. A brief introduction is given here.

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There exists roughly three approaches to vehicle formation control in the literature: leader-following, behavioral methods and virtual structures.

Briefly explained, the leader-following architecture defines a leader in the formation while the other members of the formation follows that leader. The behavioral approach prescribes a set of desired behaviors for each member in the group, and weight them such that desirable group behavior emerges. Possible behaviors include trajectory and neighbor tracking, collision and obstacle avoidance, and formation keeping.

In the virtual structure approach, the entire formation is treated as a single, virtual, structure. Virtual structures have been achieved by for example, having all members of the formation tracking assigned nodes which move through space in the desired configuration, and using formation feedback to prevent members leaving the formation [23]. In [5] each member of the formation tracks a virtual element, while the motion of the elements are governed by a formation function that specifies the desired geometry of the formation.

Mechanical constraint forces which cause independent bodies to act in accordance with given geometric constraints, are well known from the early days of analytical mechanics [16] and has been used with success, e.g. in computer graphics applications [1], [2]. The main idea in this paper is to show how classic and powerful tools from analytical mechanics for multi-body dynamics [17] can be used for formation control issues. A collection of independent bodies/vehicles can be controlled as a virtual structure by introducing functions that describe a vehicles behavior with respect to the others. In this setting, imposing constraints on a system lead to control laws, which can be combined with the formation constraints that maintain the virtual structure, and together form control laws that govern the movement of the entire formation.

The rest of this paper is organized as follows. In Section II, we model a system with constraints, present stabilizing methods, and show how constraints can be imposed for control purposes. Section III presents examples of formation control of marine craft with constraints, and Section IV contains some concluding remarks.

II. Modeling, Control, and Constraints

A. Motivating Example

To introduce the main idea of the method given in this section and illustrate how the constraint function affects the motion of independent systems, we look at a formation of two point masses $q_1, q_2 \in \mathbb{R}^2$ with kinetic energy $\mathcal{T} = \frac{1}{2}\dot{q}^\top M \dot{q}$, where $q = [q_1^\top, q_2^\top]^\top$ and $M = M^\top > 0$ is the mass matrix; $M = \mathrm{diag}\,(m_1 I_2, m_2 I_2)$. The distance

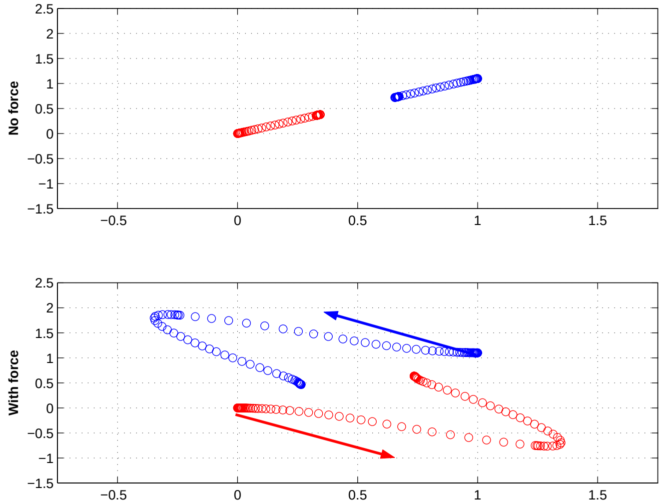

Fig. 1. Position plot of q_1 and q_2 – starts in $(0, 0)$ and $(1, 1)$ respectively. $r = 0.5$.The lower part includes the initial force disturbances.

between the point masses shall satisfy the constraint function

$$\mathcal{C}(q) = (q_1 - q_2)^\top (q_1 - q_2) - r^2 = 0 \tag{1}$$

where $r > 0$ is the desired distance between q_1 and q_2. The procedure in the Section II-B and II-C gives the following equations of motion (on the manifold where (1) is satisified)

$$M\ddot{q} = -W(q)^\top \lambda \tag{2}$$

where $W(q)$ is the Jacobian of the constraint function and λ is the Lagrangian multiplier with stabilizing feedback from the constraints.

The system has initial conditions that violate the constraint (1), but the masses converge such that (1) is met. The system converges to a position between the initial position of q_1 and q_2 where the distance between q_1 and q_2 is r. If we apply some damping and initially perturb both integrators with a small amount of force, the motion of the integrators will still satisfy the constraint function, eventually. As seen in Figure 1 the two points move such that (1) is satisfied in both cases – they *assemble* into a formation defined by the constraint function.

B. System Modeling

Consider n systems of order m with kinetic and potential energy, $\mathcal{T}_i$ and $\mathcal{U}_i$, respectively. The Lagrangian of the total system is then

$$\mathcal{L} = \mathcal{T} - \mathcal{U} = \sum_{i=1}^{n} \mathcal{T}_i - \mathcal{U}_i.$$

Suppose there exists kinematic relations

$$\mathcal{C}(q) = 0,\ \mathcal{C}(q) \in \mathbb{R}^p \tag{3}$$

between the coordinates which restricts the state space to a constraint manifold $\mathcal{M}_c$ with less than $2n \cdot m$ dimensions. We denote $\mathcal{C}(q)$ the *constraint function*, where $q \in \mathbb{R}^{nm}$ contains all generalized positions, q_i, of the n systems. We know, from [17], that the forces that maintain the kinematic constraints adds potential energy to the system, so we end up with the following, modified, Lagrangian

$$\bar{\mathcal{L}} = \mathcal{T} - \mathcal{U} + \lambda \mathcal{C}(q)$$

where λ is the Lagrangian multiplier(s). To obtain the equations of motion, we apply the Euler-Lagrange differential equations with auxiliary conditions for $i = 1, \dots, nm$,

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} + \lambda \frac{\partial \mathcal{C}(q)}{\partial q_i} + \frac{\partial \lambda}{\partial q_i}\mathcal{C}(q) = \tau_i$$

which implies

$$\begin{aligned} \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} - \frac{\partial \mathcal{L}}{\partial q_i} + \lambda \frac{\partial \mathcal{C}(q)}{\partial q_i} &= \tau_i \\ \mathcal{C}(q) &= 0 \end{aligned} \tag{4}$$

where τ_i is the generalized external force associated with coordinate q_i.

Equation (3) constrains the systems motion to a subset, $\mathcal{M}_c \subseteq \mathbb{R}^{2nm-p}$, of the state space where $\mathcal{C}(q) = 0$. Since we want to keep the systems on $\mathcal{M}_c$, neither the velocity nor the acceleration should violate the constraints. This gives the additional conditions

$$\begin{aligned} \dot{\mathcal{C}}(q) &= W(q)\dot{q} = 0 \\ \ddot{\mathcal{C}}(q) &= W(q)\ddot{q} + \dot{W}(q)\dot{q} = 0 \end{aligned} \tag{5}$$

where $W(q) \in \mathbb{R}^{mn \times p}$ is the Jacobian of the constraint function, i.e. $W(q) = \frac{\partial \mathcal{C}(q)}{\partial q}$. We combine (4) and (5) to obtain an expression for the Lagrangian multiplier.

The main focus in this paper is marine craft, so we introduce the equations of motion for a marine vessel in the body-fixed frame, derived analytically in [7] using an energy approach,

$$\begin{aligned} \dot{\eta} &= R(\psi)\nu \\ M\dot{\nu} + C(\nu)\dot{\nu} + D(\nu)\nu + g(\eta) &= \tau \end{aligned}$$

where $\eta = [x, y, \psi]^\top$ is the Earth-fixed position vector, (x, y) is the position on the ocean surface and ψ is the heading angle (yaw), and $\nu = [u, v, r]^\top$ is the body-fixed velocity vector. The model matrices M, C, and D denote inertia, Coriolis plus centrifugal and damping, respectively, while g is a vector of generalized forces and $R = R(\psi) \in SO(3)$ is the rotation matrix between the body and Earth coordinate frame[1]. For more details regarding ship modeling, the reader is suggested to consult [7] and [10]. Consider a formation of n vessels with position given by η_i, inertia matrix M_i, and so on. We collect the vectors into new vectors, and the matrices into new, block-diagonal, matrices by defining $\eta = [\eta_1^\top, \dots, \eta_n^\top]^\top$, $M = \mathrm{diag}\{M_1, \dots, M_n\}$, and so on. The addition of the potential energy from the constraints gives

$$\begin{aligned} \dot{\eta} &= R(\psi)\nu \\ M\dot{\nu} + C(\nu)\dot{\nu} + D(\nu)\nu + g(\eta) &= \tau + \tau_{\text{constraint}} \end{aligned}$$

where $\tau_{\text{constraint}} = -W(\eta)^\top \lambda$ is the expression for the constraint forces which maintain the kinematic constraint. The equations of motion can be transformed to the Earth-fixed frame by the kinematic transformation in [7, Ch.

[1]Note that this approach is also valid for mechnical systems such as $M(q)\ddot{q} + C(q, \dot{q})\dot{q} = \tau$ (robot manipulator).

3.3.1] which results in

$$M_\eta(\eta)\ddot{\eta} + n(\nu,\eta,\dot{\eta}) = \tau_\eta - R(\psi)W(\eta)^\top\lambda \quad (6)$$

where $n(\nu,\eta,\dot{\eta}) = C_\eta(\nu,\eta)\dot{\eta} + D_\eta(\nu,\eta)\dot{\eta} + g_\eta(\eta)$ and $\tau_\eta = R(\psi)\tau$.

By replacing q with η, the combination of (5) and (6) gives a differential algebraic equation (DAE). Solving for $\ddot{\eta}$ and substituting gives

$$W(\eta)M_\eta^{-1}(\eta)R(\psi)W(\eta)^\top\lambda = W(\eta)M_\eta^{-1}(\eta)\{\tau_\eta - n(\nu,\eta,\dot{\eta})\} + \dot{W}(\eta)\dot{\eta}.$$

Then, by using λ in (6), we ontain the equations of motions for the systems subject to the constraint (3).

Assumption A1: The mass matrix M is positive definite, i.e. $M = M^\top > 0$, hence $M_\eta(\eta) = M_\eta(\eta)^\top > 0$ by [7].

Assumption A2: The Jacobian $W(\eta)$ has full rank, i.e., the constraints are not conflicting or redundant, and is bounded by a linear growth rate, $k_1|\eta| \leq W \leq k_2|\eta|$, for some positive $k_{1,2} \in \mathbb{R}$. Note that redundant or conflicting constraints arise when one, or more, row (column) in $\mathcal{C}$ is a linear combination of other rows (columns), or when the functions are contradicting.

Assumptions A1 and A2 guarantees that $WM_\eta^{-1}RW$ exists since M_η is positive definite, hence M_η^{-1} exists and $WM_\eta^{-1}RW^\top$ is nonsingular.

C. Stabilization of constraints

If the system starts on the constraint manifold $\mathcal{M}_c$, that is, the initial conditions $(\eta(0),\dot{\eta}(0)) = (\eta_0,\dot{\eta}_0)$ satisfy $(\eta_0,\dot{\eta}_0) \in \mathcal{M}_c$ such that

$$\mathcal{C}(\eta_0) = 0 \text{ and } \dot{\mathcal{C}}(\eta_0) = 0$$

and the force τ does not perturb the system so that $(\eta,\dot{\eta})$ leaves $\mathcal{M}_c$, then the system is well-behaved and $(\eta,\dot{\eta}) \in \mathcal{M}_c$ for all times. However, if the initial conditions are not in $\mathcal{M}_c$, or the system is perturbed s.t. $(\eta,\dot{\eta}) \notin \mathcal{M}_c$, feedback must be used to stabilize the constraint.

We want to investigate stability of the constraint, and in terms of set-stability we look at stability of the set

$$\mathcal{M}_c = \{(\eta,\dot{\eta}) : \mathcal{C}(\eta) = 0,\ W(\eta)\dot{\eta} = 0\}.$$

Consider the case when $\tau \neq 0$ in (6), and suppose that $(\eta_0,\dot{\eta}_0) \notin \mathcal{M}_c$. If we use the equations from Section II-B, then

$$\ddot{\mathcal{C}}(\eta) = 0 \quad (7)$$

which is unstable – if $\mathcal{C}(\eta)$ is a scalar function it contains two poles at the origin. Hence, if $\mathcal{C}(\eta) = 0$ is not fulfilled initially, the solution might blow up in finite time. Even with $\mathcal{C}(\eta_0) = 0$, this might happen if there is measurement noise on η. This unstability is, in fact, an inherent property of higher-index DAEs [31].

However, if we introduce feedback from the constraints in the expression for the Lagrangian multiplier,

$$W(\eta)M_\eta^{-1}(\eta)R(\psi)W(\eta)^\top\lambda = W(\eta)M_\eta^{-1}(\eta)\{\tau_\eta - n(\nu,\eta,\dot{\eta})\} + \dot{W}(\eta)\dot{\eta} + K_d\dot{\mathcal{C}}(q) + K_p\mathcal{C}(q) \quad (8)$$

where $K_p,\ K_d \in \mathbb{R}^{p\times p}$ and positive definite, we stabilize the constraint function

$$\ddot{\mathcal{C}} = -K_d\dot{\mathcal{C}}(\eta) - K_p\mathcal{C}(\eta). \quad (9)$$

We rewrite (9), using

$$\phi_1 = \mathcal{C}(\eta), \quad \phi_2 = \dot{\mathcal{C}}(\eta), \quad \phi_3 = \ddot{\mathcal{C}}(\eta), \quad \phi = \begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix}$$

such that

$$\begin{aligned}\begin{bmatrix}\dot{\phi}_1\\ \dot{\phi}_2\end{bmatrix} &= \begin{bmatrix}0 & I\\ -K_p & -K_d\end{bmatrix}\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix} \\ \dot{\phi} &= A\phi, \qquad A \in \mathbb{R}^{2p\times 2p}\end{aligned} \quad (10)$$

By appropriate choice of K_p and K_d, A is Hurwitz. Then, by choosing a design matrix $Q = Q^\top > 0$, we can find a $P = P^\top > 0$ such that

$$PA + A^\top P = -Q.$$

Then

$$V(\phi) = \phi^\top P\phi > 0, \quad \forall\phi \neq 0 \quad (11)$$

$$\dot{V}(\phi) = -\phi^\top Q\phi < 0, \quad \forall\phi \neq 0 \quad (12)$$

Theorem 1: The set $\mathcal{M}_c$ is a globally exponentially stable (GES) set of equilibrium points of the system

$$\begin{aligned}M_\eta(\eta)\ddot{\eta} + n(\nu,\eta,\dot{\eta}) &= \tau_\eta - R(\psi)J(\eta)^\top\lambda \\ \mathcal{C}(\eta) &= 0\end{aligned} \quad (13)$$

under Assumptions A1 and A2.

Proof: The proof follows from (11) and (12). ■

By applying feedback from the constraints and converting (7) to (9) the condition (9) has become more robust, and is stable in the cases when the initial values do not fulfill the constraint and when there is measurement noise present.

Remark 1: Consider the case in the motivating example and set $K_p = \beta^2$ and $K_d = 2\alpha$, so $\ddot{\mathcal{C}} + 2\alpha\dot{\mathcal{C}} + \beta^2\mathcal{C} = 0$, $\alpha,\ \beta > 0$. In the numerical scientific society the last equation is referred to as the Baumgarte stabilization technique [3] – an algortihm used for numerical stabilization in simulations of multi-body and constrained systems.

Remark 2: Given a position control law $\tau_{\eta 1}$ for a single vessel that renders the equilibrium points of the system $M_{\eta,1}(\eta_1)\ddot{\eta}_1 + n(\nu_1,\eta_1,\dot{\eta}_1) = \tau_{\eta,1}$ UGAS. In combination with constraint forces that maintain the configuration, $M_\eta(\eta)\ddot{\eta} + n(\nu,\eta,\dot{\eta}) = \tau_\eta - R(\psi)W(\eta)^\top\lambda$, the total, closed-loop, system can be treated as a cascade – see Figure 2. Further, the constraint stabilization dynamics can be shown to be Input-to-State Stable [29] with respect to the position errors in the control law $\tau_{\eta 1}$. This implies that the system's equilibrium points remain UGAS, and the combination of the formation constraint stabilization and the position control laws yields a system that moves according to both controllers. This is a topic of ongoing research.

D. Using Constraints for Control

So far we have talked about systems with constraints without saying anything about how the constraints arise. In the control literature, the main focus has been on constrained robot manipulators with physical contact between the end effector and a constraint surface. This occurs

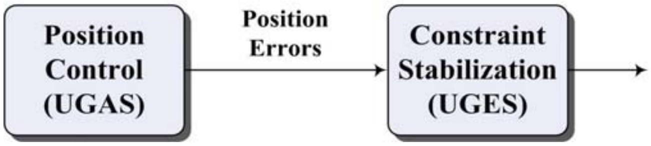

Fig. 2. Combination of constraints and control law as a cascade.

in many tasks, including scribing, writing, grinding, and others as described in [13], [18] and references therein. These constraints are inherently in the system as they are all based on how the model or environment constrain the system.

However, if *constraints are imposed* on the system, the framework for stabilization of constraints described in Section II-C can be used to design control laws with feedback from the constraints applicable for a wide variety of purposes.

Example 1: Consider a single, unforced, double integrator subject to the constraint $\mathcal{C}(q) = q = 0,\ q \in \mathbb{R}$

$$m\ddot{q} = -W(q)\lambda$$

where $W(q) = 1,\ m = 1$, and λ is given as in (8):

$$\lambda = 2\alpha\dot{q} + \beta^2 q, \qquad \alpha, \beta \in \mathbb{R}.$$

The constraint manifold is now the origin, i.e. $\mathcal{M}_c = \{(0,0)\}$. The dynamics are

$$\ddot{q} = -2\alpha\dot{q} - \beta^2 q$$

which can be shown to be GES for $\alpha, \beta > 0$. Given an inital condition $q(0) = q_0 \notin \mathcal{M}_c$, the trajectory $q(t)$ converges exponentially to the origin. The constraint function corresponds in this case to a proportional-derivative (PD) controller.

III. Case Studies

To illustrate how constraints can be used in a formation setup control scheme, we will compare some schemes from formation [28] and synchronization [15] control of marine craft and see how constraints are used implicitly. Secondly, we will focus on how a set of constraints can help us to control a formation during the initial phase – the *formation assembling phase*. Further, we will show that by using constraints to maintain a formation of vessels, we can use a path following design for one vessel to achieve formation control.

A. Control Plant Ship Model

The control plant model of a supply vessel is used in the case study [9]. We consider a vessel model for low-speed applications (up to 2-3 m/s) and station-keeping where the kinetics can be described accurately by a linear model. Extension to maneuvering at higher speeds can be done using the nonlinear model in [8]. Furthermore, the surge mode is decoupled from the sway and yaw mode in the ship model. The body-fixed equations of motion are given as

$$\dot{\eta}_i = R(\psi_i)\nu_i \qquad (14a)$$

$$M_i\dot{\nu}_i + D_i(\nu_i)\nu_i = \tau_i \qquad (14b)$$

where $D_i(\nu_i) = D_i + D_{n,i}(\nu_i)$ and M_i, D_i are nondimensional system matrices from [9] which are Bis-scaled coefficients, identified by full-scale sea trials in the North Sea. The matrix $D_n(\nu)$ contains the nonlinear damping term in surge, i.e., $D_{n,i}(\nu) = \mathrm{diag}\left(-X_{u|u|}|u|, 0, 0\right)$. The model will be transformed corresponding to Section II-B.

B. Formation Control Schemes and Constraints

This section will show that, under some assumptions, constraints of the form dicussed in this paper appear implicitly in some previous schemes for coordinated control of a group of ships.

In the formation maneuvering design given in [28] – based on the maneuvering design in [27] – the control objective is to make an error vector z go to zero – in particular, the design establishes UGES of the set $\mathcal{M} = \{(z, \theta, t) : z = 0\}$. The error vector contains information about both position and velocity, where the position error is defined as $z_{1i} = \eta_i - \xi_i$,for $i = 1, 2$, where $\eta_{1i} \in \mathbb{R}^n$ is the position and $\xi_i \in \mathbb{R}^n$ is the desired location for the i'th member of the formation. We consider a formation with two members where $\xi_i = \xi + R(\psi(\theta))l_i$, $l_i \in \mathbb{R}^3$ and we assume that $R(\psi) = I$. This corresponds to a desired motion where the formation moves parallel to the x-axis in the inertial frame. When the systems have reached $\mathcal{M}$, $z = 0$ and the position errors are $z_{1i} = \eta_i - \xi_i = \eta_i - \xi - l_i = 0$.

Combining z_{11} and z_{12}, we get $\eta_1 - \xi - l_1 = \eta_2 - \xi - l_2$, or $\eta_1 - \eta_2 - (l_1 - l_2) = \eta_1 - \eta_2 - r_{12} = 0$. Similar with the velocity errors on $\mathcal{M}$, $z_{2i} = \dot{\eta}_i - \alpha_{1i}$, where α_{1i} is a virtual control law. For the two systems we get $z_{2i} = \dot{\eta}_i - A_{1i}(\eta_i - \xi - l_i) = 0$, where $\dot{\eta}_i$ is the velocity and A_{1i} is a control design matrix. A combination of z_{21} and z_{22} gives, assuming that $A_{11} = A_{12}$, $\dot{\eta}_1 - \dot{\eta}_2 = A_{11}(z_{11} - z_{12}) = 0$.

Hence, on $\mathcal{M}$, the error variable gives constraints on the form

$$\begin{aligned} \mathcal{C}(x) &= \eta_1 - \eta_2 - r_{12} = 0 \\ \dot{\mathcal{C}}(x) &= \dot{\eta}_1 - \dot{\eta}_2 = W\dot{\eta} = 0. \end{aligned}$$

With the above assumptions we see that in the formation assembling phase, the set $\mathcal{M}$ corresponds to $\mathcal{M}_c$.

In [15] the authors use synchronization techniques to develop a control law for rendezvous control of ships. In a case study with two ships the control objective is to control the supply ship to a position relative to the main ship. The desired configuration is reached when the errors $e = \eta_S - \eta_M$ and $\dot{e} = \dot{\eta}_S - \dot{\eta}_M$ are zero, where the subscripts S and M stands for supply- and main ship, respectively. Both error functions fit into the framework for constraints in Section II-B.

For a replenishment operation, we can define the constraint to depend on the lateral position coordinate only. Then, the supply vessel converge to a position paralell to the course of the main ship, and this position can be maintained during forward speed for replenishment purposes.

C. Case 1: Assembling of Marine Craft

We consider a formation of three vessels where the control objective is to assemble the craft into a predefined configuration, e.g. in order to be in position to tow a barge or another object. We assume there is no external force or control law acting on the formation, i.e. $\tau = 0$, and the purpose is to show that assembling of the individual vessels into a formation can be done by imposing constraints.

Consider the constraint function

$$\mathcal{C}_1(\eta) = \begin{bmatrix} (\eta_1^* - \eta_2^*)^\top (\eta_1^* - \eta_2^*) - r_{12}^2 \\ (\eta_2^* - \eta_3^*)^\top (\eta_2^* - \eta_3^*) - r_{23}^2 \\ (\eta_3^* - \eta_1^*)^\top (\eta_3^* - \eta_1^*) - r_{31}^2 \end{bmatrix} = 0 \qquad (15)$$

where $\eta_i^* \in \mathbb{R}^2$ is the reduced position vector without the orientation ψ and $r_{ij} \in \mathbb{R}$ is the distance between vessel i and j. This constraint enables us to specify the positions of each vessel with respect to the others. The constraint manifold is now equivalent to the formation configuration, and by the previous sections we know that we can stabilize $\mathcal{M}_c$ by using feedback from the constraints. Hence, the constraints method gives control laws for formation assembling. From the ship model and the constraint, we have

$$M_\eta \ddot{\eta} + D_\eta \dot{\eta} = -R(\psi) W(\eta)^\top \lambda$$

where $M_\eta = M_\eta(\eta) = \operatorname{diag}(M_{\eta 1}, M_{\eta 2}, M_{\eta 3})$, $D_\eta = D_\eta(\nu, \eta) = \operatorname{diag}(D_{\eta 1}, D_{\eta 2}, D_{\eta 3})$, $\eta = \left[\eta_1^\top, \eta_2^\top, \eta_3^\top\right]^\top$, and, so on. The Lagrangian multiplier is obtained from

$$W M_\eta^{-1} R W^\top \lambda = -W M_\eta^{-1} D_\eta \dot{\eta} + \dot{W}(\eta)\dot{\eta} + K_d \dot{\mathcal{C}}(q) + K_p \mathcal{C}(q).$$

Equation (15) gives the configuration of the formation, but does not provide any information about location. If the control objective is to assemble the formation and position the first vessel in a desired location, we can add a row to (15), such that

$$\mathcal{C}_2(\eta) = \begin{bmatrix} (\eta_1^* - \eta_2^*)^\top (\eta_1^* - \eta_2^*) - r_{12}^2 \\ (\eta_2^* - \eta_3^*)^\top (\eta_2^* - \eta_3^*) - r_{23}^2 \\ (\eta_3^* - \eta_1^*)^\top (\eta_3^* - \eta_1^*) - r_{31}^2 \\ \eta_1 - \eta_{\text{des}} \end{bmatrix} = 0 \qquad (16)$$

where $\eta_{\text{des}} \in \mathbb{R}^3$ is the fixed desired position and orientation for the first vessel. The closed-loop equations have the same structure as before, except that $\mathcal{C}_1$ is replaced with $\mathcal{C}_2$. The last addition in the constraint function is equal to a PD-controller for the first vessel, so (16) leads to a combination of a formation controller and a PD-type-controller for dynamic positioning of ships [7]. Hence, several control laws can be handled in one step using the constraint approach.

The control parameters chosen to stabilize the formation constraints $\mathcal{C}_1$ and $\mathcal{C}_2$ are ($I = I_{3\times 3}$) $K_p = 0.8I$, $K_d = 0.8I$, the formation is defined by $r_{12} = 3$, $r_{23} = 3$, $r_{31} = 3$, and the desired position for the first vessel is $\eta_{\text{des}} = [10, 5, 0]^\top$. The vessels starts in $\eta_{10} = [8, 8, 0]^\top$, $\eta_{20} = [-2, 2, 0]^\top$, and $\eta_{30} = [-2, -2, 0]^\top$ – all with zero initial velocity.

In Figure 3 the time-plot of the constraint function $\mathcal{C}_1$ and its time-derivative, $W_1(\eta)\dot{\eta}$, are shown. The constraints and velocity terms converge to zero, and the constraint manifold is reached. The vessels have converged to the nearest positions where the constraints are fulfilled, and the formation is assembled in the desired configuration, as seen in Figure 4.

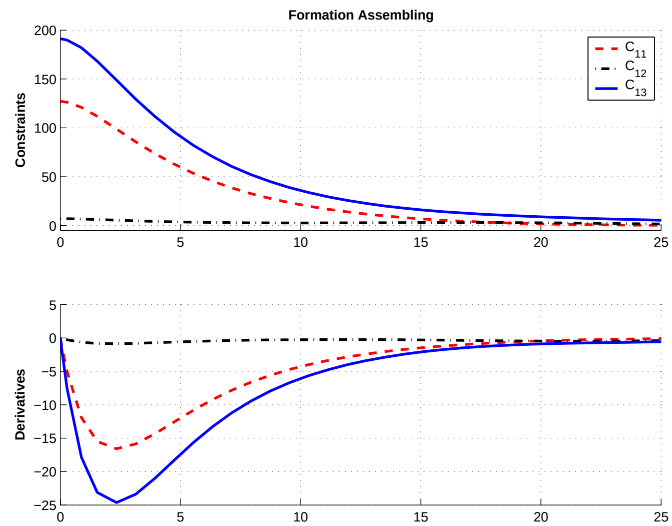

Fig. 3. Time response of formation constraints during assembling. $\mathcal{C}_{1i}$ corresponds to the i'th row in $\mathcal{C}_1$.

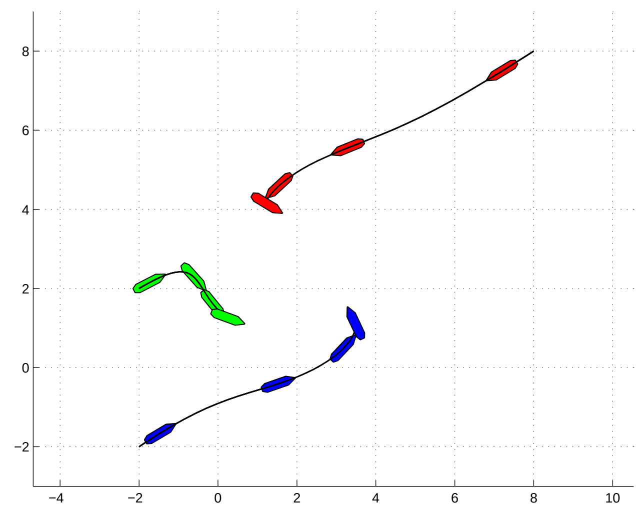

Fig. 4. Position response of vessels during assembling.

Figure 5 shows the position of the three vessels with the same inital conditions as before but subject to the constraint $\mathcal{C}_2$. The vessels assemble according to the constraints, but this time vessel 1 is positioned at its desired position η_{des}. This forces the two other vessels to move to a different position compared to the first case in order to satify the constraint. The additional constraint slows down convergence to the constraint manifold since vessel 1 has to be positioned at η_{des} and this forces the other vessels to move such that the constraint is fulfilled.

D. *Case 2: Maneuvering a formation*

In the previous section, we showed that by imposing constraints a formation can be assembled at a desired location. Centralized and decentralized formation maneuvering has been achieved in [28], [12], respectively, where each

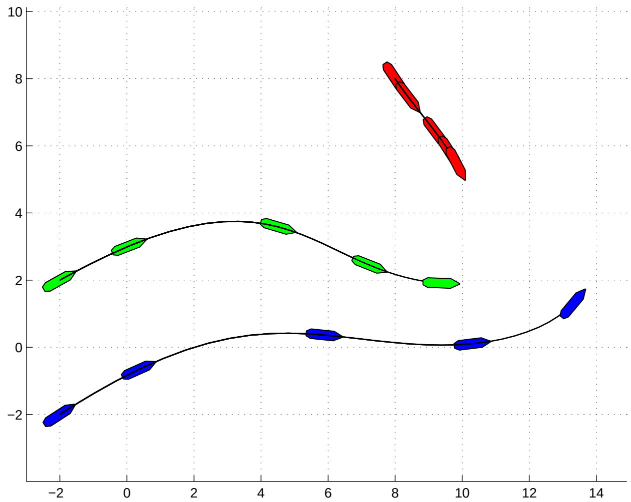

Fig. 5. Position response of vessels subject to formation constraints when vessel 1 is to be positioned at η_{des}.

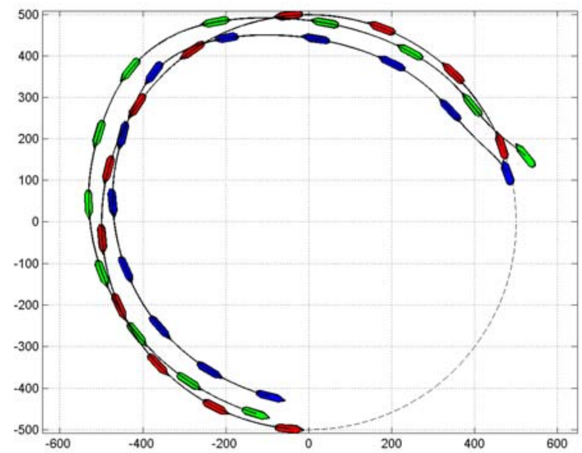

Fig. 6. Position plots of three vessels in a formation where vessel 1 (red) is following a desired path (dashed). The resulting vessel trajectories are shown as solid lines.

member of the formation follows a predefined path. To show that constraints also can be used to maintain the structure of a formation during movement, we will use a path following scheme for one of the vessel, and apply a constraint function to assure that the distance between the vessels satisfies the formation configuration. We use the constraint function $\mathcal{C}_1(\eta)$ from (15). This constraint is used since we want only vessel 1 to follow a predefined path – the other vessels will follow while maintaining the distance.

We have the closed-loop system

$$M_\eta \ddot{\eta} + D_\eta \dot{\eta} = \tau_\eta - R(\psi) J(\eta)^\top \lambda$$

where

$$JM_\eta^{-1} R J^\top \lambda = JM_\eta^{-1}(\tau_\eta - D_\eta \dot{\eta}) + \dot{J}(\eta)\dot{\eta} + K_d \dot{\mathcal{C}}(q) + K_p \mathcal{C}(q),$$

and the maneuvering design of the nominal system gives us the following signals, $\tau_\eta = [\tau_{\eta 1}, 0, 0]^\top$,

$$\begin{aligned}
&z_1 := \eta_1 - \xi(\theta), \ z_2 := \dot{\eta}_1 - \alpha_1 \\
&\alpha_1 = R_1(\psi)^\top \left[A_1 z_1 + \xi^\theta(\theta)\, \upsilon(\theta, t)\right] \\
&\sigma_1 = \dot{R}_1^\top R_1 \alpha_1 + R_1^\top \left[A_1 R_1 \upsilon_s + \xi^\theta \upsilon_s^t\right] \\
&\alpha_1^\theta = R_1^\top \left[-A_1 \xi^\theta + \xi^{\theta^2} \upsilon_s + \xi^\theta \upsilon_s^\theta\right] \\
&\gamma_2 = -2 z_1^\top P_1 \xi^\theta - 2 z_2 P_2 \alpha_1^\theta \\
&\tau_1 = M_{\eta 1}[-P_2^{-1} R_1(\psi)^\top P_1 z_1 + A_2 z_2 \\
&+ M_{\eta 1}^{-1} D_{\eta 1} \nu_1 + \sigma_1 + \alpha_1^\theta \upsilon_s] \\
&\dot{\theta} = \upsilon_s - \mu \gamma_2
\end{aligned}$$

where $\xi(\theta)$ is the desired path, $\upsilon_s(\theta, t)$ is a speed assignment, and the other signals come from the backstepping design [27]. Notice that it is possible to show that the combination of constraints and maneuvering controller is UGAS as discussed in Remark 2.

The desired path is defined as a circle with radius $r = 500$

$$\xi(\theta) = \begin{bmatrix} x_d(\theta) \\ y_d(\theta) \\ \psi_d(\theta) \end{bmatrix} = \begin{bmatrix} r\cos\left(\frac{\theta}{r}\right) \\ r\sin\left(\frac{\theta}{r}\right) \\ \operatorname{atan2}\left(\frac{y_d^\theta(\theta)}{x_d^\theta(\theta)}\right) \end{bmatrix}$$

The control parameters in the maneuvering part are set as ($I = I_{3\times 3}$) $A_1 = -0.5I$, $A_2 = -\operatorname{diag}(2, 2, 20)$, $P_1 = 0.6I$, $P_2 = \operatorname{diag}(10, 10, 40)$, and $\mu = 10$, while $K_p = I$ and $K_d = I$. The configuration of the formation is defined by $r_{12} = 90$, $r_{23} = 60$, $r_{31} = 90$, and the initial conditions for the vessels are $\eta_1(0) = [500, 0, 0]^\top$, $\eta_2(0) = [445, 63, 0]^\top$, $\eta_3(0) = [549, 29, 0]$, $\dot{\eta}_{1,2,3}(0) = 0$ and $\theta(0) = 0$. The speed assignment for Vessel 1, υ_s, is chosen correpsonding to a desired surge speed of 2 m/s along the path.

Figure 6 shows that Vessel 1 follows the desired path accurately, and the formation stays together throughout the simulation. The distance between the vessels is constant (when the states are on the constraint manifold), so the control objective is satisfied and even though Vessel 2 and 3 do not follow predefined paths, the constraints prevent collisions by maintaining the formation configuration. The constraints during the first 25 seconds of the simulation – Figure 7 – shows that after the constraints converge to zero the states remain on $\mathcal{M}_c$.

E. Discussions

This section have only considered constraint functions that leads to a triangular formation, but depending on the number of members in the formation, the formation can be configured in many different ways, e.g. line- (in the transversal or longitudinal direction), circle- or box-shaped, by changing the constraint function. Different formation configurations can be useful during changing operations, such as collision avoidance, sea-bed scanning, etc.

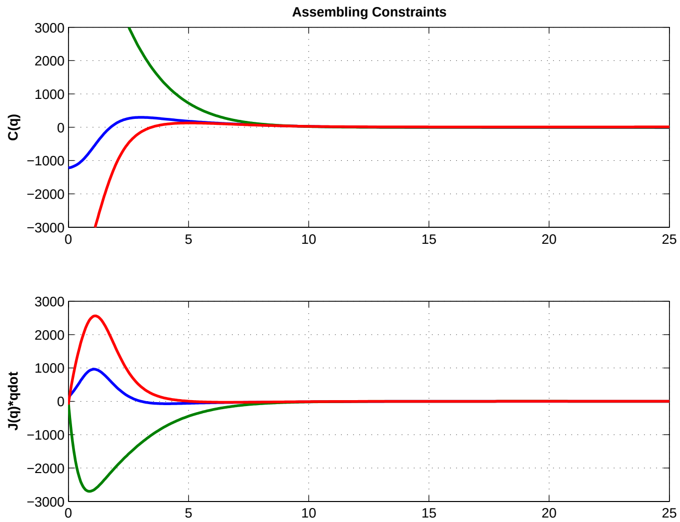

Fig. 7. Time-plot of constraints.

For simplicity, we have only considered holonomic, inter-vessel, constraints in this paper, i.e., kinematic constraints which can be expressed as finite relations between the generalized coordinates, but the proposed method can be extended to nonholonomic constraints which are linear in the generalized velocities [11].

IV. Conclusion

In this paper, we have proposed a new method for constructing control laws based on imposing constraints functions which again impose forces that maintain the constraints. In particular, we have come up with control laws which maintain a formation structure subject to measurement errors, poor initialization, and forced motion. Furthermore, we have seen how constraints appear in many control schemes for multiple marine craft, and how our approach can be used in combination with existing control laws in order to simultaneously achieve some desired behavior and maintain formation configuration. Applications of this method have been illustrated by simulations of formation of marine craft.

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