# Output Feedback Control for Maneuvering Systems Using Observer Backstepping

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***Abstract*— An output feedback design for maneuvering systems is proposed by using an observer backstepping approach which applies damping terms to counteract disturbances to the controller. By using available exponential observers, stability of the interconnected observer-controller system is obtained. The design is fairly general, and equally applicable for maneuvering systems where an exponentially stable observer is available. As a case study, an output-feedback control design is performed for maneuvering a vessel where only position measurements are available. The resulting design can be viewed as a Guidance-Navigation-Control system for a marine vessel. Simulation of the case study verifies the theoretical results.**

## I. Introduction

Control systems which primarily wants to steer an object along a path, while the speed assignment may be of secondary interest, is referred to as maneuvering systems. One advantage of following a path without just considering time assignments, is that a dynamic combination of time, speed and acceleration assignments and feedback from the plant can be used to determine the desired motion along the path. For a thorough background on trajectory tracking, path following, and maneuvering, see [1], [2] and references therein.

The general maneuvering problem, as stated in [2], divides the control problem into a geometric and a dynamic task: the geometric task is to reach the desired path and then stay on it, while the dynamic task is to satisfy a time, speed or acceleration assignment along the path.

The obtained control law, when implemented as a real life control system, may depend on states that are not measured. An observer is therefore designed to estimate the unknown states needed in the control law. For linear systems, the superposition principle guarantees that two stable subsystems will remain stable when interconnected as a cascade. For nonlinear systems, output-feedback introduces new challenges: nonlinear observers are only available for restrictive classes of systems, and the interconnection of two nonlinear, stable, systems might not be stable. See [3] and references therein for an overview. References on observer design and output feedback control of dynamically positioned ships can be found in, e.g., [4], [5], [6] and [7].

In this paper an output-feedback design method for maneuvering systems is proposed. State-feedback control design is used for solving a robust output maneuvering problem in [2], adaptive output maneuvering in [8], and formation control by synchronizing multiple maneuvering systems in [9]. Experimental results on maneuvering and formation maneuvering systems are reported in [8], [10]. However, no complete output-feedback analysis is given. This paper is motivated by the observer backstepping approach from [11] where damping terms are added to the controller to counteract the disturbances from the exponentially stable observer, and thereby ensure stability of the closed-loop system. This approach can be seen as an extension to the general maneuvering problem, since the difference between the state-feedback control system and the output-feedback control system lies in the addition of the damping term.

In Section II, an output-feedback maneuvering design for a class of systems on output-feedback form will be performed. The same output-feedback control design is applied for a surface vessel with only position measurements in Section III. Simulation results is used to verify the theoretical results of the proposed design in Section IV, and some concluding remarks are given in V.

**Notation:** Abbreviations like GS, LAS, UGAS, UGES etc, are U for Uniform, G for Global, L for Local, A for Asymptotically, E for Exponential and S for Stable. A superscript denotes partial differentiation: $f^{x}(x,y) := \frac{\partial f}{\partial x}$, $f^{x^2}(x,y) := \frac{\partial^2 f}{\partial x^2}$, where the gradient $f^x$ is a row vector. The Euclidean vector norm is $|x| := \left(x^\top x\right)^{1/2}$, the distance to the set $\mathcal{M}$ is $|x|_{\mathcal{M}} := \inf_{y\in\mathcal{M}} |x-y|$, and the induced matrix 2-norm of $A \in \mathbb{R}^{n\times n}$ is denoted $\|A\|$. For a matrix $P = P^\top > 0$, let $p_m := \lambda_{\min}(P)$ and $p_M := \lambda_{\max}(P)$.

### A. Problem statement

The objective of the proposed output-feedback design is to solve the maneuvering problem [1], [2] where an observer estimates the unknown states. The considered maneuvering problem consists of two tasks. The first task, the geometric task, is for the system output to converge to and follow a desired path $y_d(\theta)$, where $\theta$ is the path variable. In the second task, the dynamic task, the path speed shall converge to a desired speed $\upsilon_s(\theta,t)$. Finally, the output of the observer shall converge to the measured states of the actual system. Collecting the tasks above, this is denoted the maneuvering problem with output feedback.

## II. Maneuvering Design for Output-feedback Systems

The control objective in this section is to solve the maneuvering problem with output feedback for the class

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of *output-feedback systems*, whose output $y$ is the only measured signal. These systems can be transformed into the *output-feedback form*, in which nonlinearities depend only on $y$. Consider the vectorial case $x_{1,2} \in \mathbb{R}^n$ and let $x := \left[x_1^\top,\ x_2^\top\right]^\top$.

$$\begin{aligned}\dot{x} &= Ax + f(y) + G(y)u \\ y &= c^\top x, \text{ where}\end{aligned}$$

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix}, f(y) = \begin{bmatrix} f_1(y) \\ f_2(y) \end{bmatrix}, G(y) = \begin{bmatrix} 0 \\ G_2(y) \end{bmatrix},$$

and $c = [I, 0]^\top$. Assume that the functions $G_2,\ f_1,\ f_2$ are smooth, and the matrix $G_2$ is invertible. Since only $x_1$ is measured, an observer that can provide information about the unknown state $x_2$ must be designed. This particular class of nonlinear systems is chosen since there exist exponentially stable observers for these systems

$$\begin{aligned}\dot{\hat{x}} &= A\hat{x} + K(y - \hat{y}) + f(y) + G(y)u \\ \hat{y} &= c^\top \hat{x}\end{aligned}$$

where $K = \left[K_1^\top, K_2^\top\right]^\top$ is chosen so that $A_o = A - kc^\top$ is Hurwitz. The error dynamics is $\dot{\tilde{x}} = A_o\tilde{x}$. Then, there exists a $P_o = P_o^\top > 0$ such that $A_o^\top P_o + P_o A_o = -I$. Note that $|\tilde{x}| \geq |\tilde{x}_i|$ for $i = 1, 2$.

**Assumption**: The path $y_d(\theta)$ and its partial derivatives, $y_d^\theta(\theta),\ y_d^{\theta^2}(\theta)$, are uniformly bounded on $\mathbb{R}^n$. The speed assignment $\upsilon_s(\theta, t)$ and its partial derivatives, $\upsilon_s^\theta(\theta, t), \upsilon_s^t(\theta, t)$, are uniformly bounded in $\theta$ and $t$.

### A. Output-feedback Control Design

Consider the system

$$\begin{aligned}\dot{x}_1 &= x_2 + f_1(y) = \hat{x}_2 + \tilde{x}_2 + f_1(y) \\ \dot{\hat{x}}_2 &= G_2(y)u + f_2(y) + K_2\tilde{y}.\end{aligned}$$

**Step 1:** Introduce the error variables

$$z_1(x_1, \theta) := y - y_d(\theta) = x_1 - y_d(\theta) \quad (1)$$

$$z_2(x, \hat{x}, \theta, t) := \hat{x}_2 - \alpha_1(x_1, \theta, t) \quad (2)$$

$$\omega_s(\theta., \theta, t) := \upsilon_s(\theta, t) - \dot{\theta} \quad (3)$$

where $\alpha_1$ is a virtual control to be specified later. Differentiate $z_1$ w.r.t. time

$$\begin{aligned}\dot{z}_1 &= \dot{y} - y_d^\theta\dot{\theta} = \hat{x}_2 + \tilde{x}_2 + f_1(y) - y_d^\theta\dot{\theta} \\ &= z_2 + \alpha_1 + \tilde{x}_2 + f_1(y) - y_d^\theta\dot{\theta}.\end{aligned}$$

Define the first control Lyapunov function (clf)

$$V_1(x_1, \tilde{x}, \theta) = z_1(x_1, \theta)^\top P_1 z_1(x_1, \theta) + \frac{1}{d_1}\tilde{x}^\top P_o\tilde{x}$$

where $d_i > 0,\ P_i = P_i^\top > 0,\ i = o, 1$, and whose time derivative is

$$\begin{aligned}\dot{V}_1 &= 2z_1^\top P_1\left(\alpha_1 + f_1(y) - y_d^\theta\upsilon_s\right) + 2z_1^\top P_1\tilde{x}_2 \\ &\quad +2z_1^\top P_1 z_2 + 2z_1^\top P_1 y_d^\theta\omega_s - \frac{2}{d_1}\tilde{x}^\top P_o A_o\tilde{x}\end{aligned}$$

Pick the virtual control law as

$$\alpha_1 = \alpha_1(x_1, \theta, t) = A_1 z_1 - f_1 + y_d^\theta\upsilon_s + \alpha_{10}.$$

The damping term $\alpha_{10}$ will be chosen later in the design process. Define the first tuning function, $\tau_1 \in \mathbb{R}$, by $\tau_1(x_1, \theta) = 2z_1^\top P_1 y_d^\theta$. An application of Young's inequality yields

$$2z_1^\top P_1\tilde{x}_2 \leq 2\kappa_1 z_1^\top P_1 P_1 z_1 + \frac{1}{2\kappa_1}\tilde{x}_2^\top\tilde{x}_2,\ \kappa_1 > 0 \quad (4)$$

so

$$\begin{aligned}&\dot{V}_1 \leq -z_1^\top Q_1 z_1 + 2z_1^\top P_1 z_2 + \tau_1\omega_s \\ &+2z_1^\top P_1\left[\alpha_{10} + \kappa_1 P_1\right]z_1 + \frac{1}{2\kappa_1}\tilde{x}_2^\top\tilde{x}_2 - \frac{1}{d_1}\tilde{x}^\top\tilde{x}.\end{aligned}$$

Pick first damping term as $\alpha_{10} = -\kappa_1 P_1$ and then

$$\dot{z}_1 = A_1 z_1 + z_2 + y_d^\theta\omega_s - \kappa_1 P_1 z_1 + \tilde{x}_2$$

$$\dot{V}_1 \leq -z_1^\top Q_1 z_1 + 2z_1^\top P_1 z_2 + \tau_1\omega_s - c_1\tilde{x}^\top\tilde{x},$$

where $c_1 = \frac{1}{d_1} - \frac{1}{2\kappa_1} > 0$. To aid the design in the next step, the virtual control law is differentiated w.r.t. time

$$\dot{\alpha}_1 = \sigma_1 + \alpha_1^\theta\dot{\theta},\ \sigma_1 = \alpha_1^{x_1}\dot{x}_1 + \alpha_1^t.$$

**Step 2:** Differentiation of $z_2$ w.r.t. time gives

$$\dot{z}_2 = \dot{\hat{x}}_2 - \dot{\alpha}_1 = G_2(y)u + f_2(y) + K_2\tilde{y} - \sigma_1 - \alpha_1^\theta\upsilon_s + \alpha_1^\theta\omega_s.$$

Define the second clf

$$V_2(x, \tilde{x}, \theta, t) = V_1 + z_2^\top P_2 z_2 + \frac{1}{d_2}\tilde{x}^\top P_o\tilde{x},\ P_2 = P_2^\top > 0 \quad (5)$$

where $d_2 > 0$, with time derivative

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + 2z_2^\top P_2\left(G_2 u + f_2 - \sigma_1 - \alpha_1^\theta\upsilon_s\right) \\ &\quad +2z_2^\top P_2\alpha_1^\theta\omega_s + 2z_2^\top P_2 K_2\tilde{y} - \frac{1}{d_2}\tilde{x}^\top\tilde{x} \\ &\leq -z_1^\top Q_1 z_1 + 2z_2^\top P_2(P_2^{-1}P_1 z_1 + G_2 u + f_2 - \sigma_1 \\ &\quad -\alpha_1^\theta\upsilon_s) + \tau_1\omega_s + 2z_2^\top P_2\alpha_1^\theta\omega_s + c_1\tilde{x}^\top\tilde{x} \\ &\quad +2z_2^\top P_2 K_2\tilde{y} - \frac{1}{d_2}\tilde{x}^\top\tilde{x}.\end{aligned}$$

Pick the final control law as

$$\begin{aligned}u &= u(x, \tilde{x}, \theta, t) = \\ &\quad G_2^{-1}\left[A_2 z_2 - P_2^{-1}P_1 z_1 - f_2 + \sigma_1 + \alpha_1^\theta\upsilon_s + u_0\right],\end{aligned}$$

and define the second tuning function $\tau_2 \in \mathbb{R}$, by $\tau_2(x, \theta) = \tau_1 + 2z_2^\top P_2\alpha_1^\theta$. Young's inequality yields

$$2z_2^\top P_2 K_2\tilde{y} \leq 2\kappa_2 z_2^\top P_2 K_2 K_2{}^\top P_2 z_2 + \frac{1}{2\kappa_2}\tilde{y}^\top\tilde{y}, \kappa_2 > 0. \quad (6)$$

Hence, $\dot{V}_2$ becomes, with $z := [z_1,\ z_2]^\top$, $P = \mathrm{diag}(P_1,\ P_2)$, and $Q := \mathrm{diag}(Q_1,\ Q_2)$,

$$\begin{aligned}\dot{V}_2 &\leq -z^\top Qz + \tau_2\omega_s + 2z_2^\top P_2\left[u_0 + \kappa_2 K_2 K_2^\top P_2\right]z_2 \\ &\quad +\frac{1}{2\kappa_2}\tilde{y}^\top\tilde{y} - \frac{1}{d_2}\tilde{x}^\top\tilde{x} + c_1\tilde{x}^\top\tilde{x}.\end{aligned}$$

Pick the second damping term as $u_0 = -\kappa_2 K_2 K_2{}^\top P_2$, and hence

$$\dot{z}_2 = A_2 z_2 - P_2^{-1}P_1 z_1 - \kappa_2 K_2 K_2{}^\top P_2 z_2 + K_2\tilde{y} + \alpha_1^\theta\omega_s.$$

Then $\dot{V}_2$ is bounded by

$$\dot{V}_2 \leq -z^\top Qz + \tau_2\omega_s - c_2\tilde{x}^\top\tilde{x},\ c_2 = c_1 + \frac{1}{d_2} - \frac{1}{2\kappa_2} > 0 \tag{7}$$

The final tuning function can be written as $\tau_2(x,\hat{x},\theta,t) = 2g^\top Pz(x,\hat{x},\theta,t)$, and the $z-$system as $\dot{z} = Fz + g\omega_s + H\tilde{y}$, where

$$\begin{aligned} F &= \begin{bmatrix} A_1 - \kappa_1 P_1 & I \\ -P_2^{-1}P_1 & A_2 - \kappa_2 K_2 K_2^\top P_2 \end{bmatrix}, \\ g &= \begin{bmatrix} y_d^\theta \\ \alpha_1^\theta \end{bmatrix},\ H = \begin{bmatrix} 0 \\ K_2 \end{bmatrix}. \end{aligned}$$

The maneuvering with output feedback problem can now be stated as rendering the set

$$\mathcal{M} = \left\{(z,\tilde{x},\theta,t) \in \mathbb{R}^{2n} \times \mathbb{R}^{2n} \times \mathbb{R} \times \mathbb{R}_{\geq 0} : z = 0,\ \tilde{x} = 0\right\}$$

UGAS under the additional assumption that the speed assignment is fulfilled in $\mathcal{M}$, $(z,\tilde{x},\theta,t) \in \mathcal{M} \Longrightarrow \omega_s = 0$. Finally, the loop is closed by speed assignment design. From [2], three different choices are available to render $\tau_2\omega_s$ negative in (7). Setting $\omega_s = 0$, the speed assignment is satisfied identically and equivalent to a trajectory tracking design with $\dot{\theta} = \upsilon_s(\theta,t)$. Incorporating state feedback, the *gradient update law* $\omega_s = -\mu\tau_2,\ \mu > 0$ gives

$$\dot{V}_2 \leq -z^\top Qz - \mu\tau_2^2 - c_2\tilde{x}^\top\tilde{x} \leq 0, \tag{8}$$

and the update law for $\theta$ becomes

$$\dot{\theta} = \upsilon_s(\theta(t),t) + \mu\tau_2.$$

*Theorem 1:* The following closed-loop system with the gradient update law

$$\begin{aligned} \dot{z} &= Fz - \mu gg^\top Pz + H\tilde{x} \\ \dot{\theta} &= \upsilon_s(\theta,t) + 2\mu g^\top Pz \\ \dot{\tilde{x}} &= A_o\tilde{x} \end{aligned} \tag{9}$$

is, under the assumptions on plant and path signals, forward complete and solves the maneuvering problem with output feedback, i.e. the system (9) is forward complete and the set

$$\mathcal{M} = \{(z,\tilde{x},\theta,t) : z = 0,\ \tilde{x} = 0\}$$

is UGAS.

*Proof:* The proposed speed assignment is satisfied in $\mathcal{M}$, since for $z = 0$ and $\tilde{x} = 0$ that $\tau_2 = 2z^\top Pz^\theta = 0$ which further implies that $(z,\tilde{x},\theta,t) \in \mathcal{M} \Longrightarrow \omega_s = 0 \Longrightarrow \dot{\theta} = \upsilon_s$ as required. Let $Z := \left[z^\top,\ \tilde{x}^\top\right]^\top$. For the Lyapunov function (5), the bounds are

$$\begin{aligned} p_1|Z|^2 &\leq V_2 \leq p_2|Z|^2 \\ \dot{V}_2 &\leq -q_m|z|^2 - c_2|\tilde{x}|^2 \leq -p_3|Z|^2 \end{aligned}$$

where $p_1 = \min(p_m,\ p_{o.m})$, $p_2 = \max(p_M,\ p_{o,M})$, and $p_3 = \min(q_m,\ c_2)$. This implies that for all $t$ in the maximal interval of definition $[0,T)$,

$$|Z(t)| \leq \sqrt{\frac{p_2}{p_1}}|Z(0)|$$

The assumed smoothness of plant dynamics and boundedness of all path and speed assignment signals implies that the right-hand side of (9) depends continuously on $(\theta,t)$ through bounded functions. With $z$ bounded it follows that (9) is bounded on the maximal interval of definition. This excludes finite escape times so $T = +\infty$, and hence

$$|(z,\tilde{x},\theta,t)|_{\mathcal{M}} = |Z|.$$

Since $\dot{V}_2 \leq -p_3|Z|^2 = -\alpha_3(|Z|),\ \alpha_3 \in \mathcal{K}$, and with $V_2$ radially unbounded and the closed-loop system forward complete, the set $\mathcal{M}$ is UGAS and the maneuvering problem with output feedback is solved. ■

Alternatively, in [2], [8] the *filtered-gradient update law*, is constructed by augmenting the second clf to

$$V = V_2 + \frac{1}{2\mu\lambda}\omega_s^2, \quad \mu,\ \lambda > 0 \tag{10}$$

whose derivative is

$$\dot{V} \leq -z^\top Qz + \left[\tau_2 + \frac{1}{\mu\lambda}\dot{\omega}_2\right]\omega_s - c_2\tilde{x}^\top\tilde{x}$$

The second term can be rendered negative by choosing the update law for $\dot{\omega}_s$ as $\dot{\omega}_s = -\lambda(\omega_s + \mu\tau_2)$, which gives

$$\dot{V} \leq -z^\top Qz - \frac{1}{\mu}\omega_s^2 - c_2\tilde{x}^\top\tilde{x} \leq 0, \tag{11}$$

and the update law for $\theta$ becomes

$$\begin{aligned} \dot{\theta} &= \upsilon_s(\theta,t) - \omega_s \\ \dot{\omega}_s &= -\lambda\omega_s - \lambda\mu\tau_2. \end{aligned}$$

Augmenting $\mathcal{M}$ with the state $\omega_s$, gives the following theorem.

*Theorem 2:* The closed-loop system with the filtered-gradient update law

$$\begin{aligned} \dot{z} &= F(\tilde{y})z + g\omega_s + H\tilde{x} \\ \dot{\theta} &= \upsilon_s(\theta,t) - \omega_s \\ \dot{\omega}_s &= -\lambda\omega_s - 2\lambda\mu g^\top Pz \\ \dot{\tilde{x}} &= A_o\tilde{x} \end{aligned} \tag{12}$$

is, under the assumptions on plant and path signals, forward complete and solves the maneuvering with output feedback problem, i.e., the system (12) is forward complete and the set

$$\mathcal{M} = \{(z,\tilde{x},\omega_s,\theta,t) : z = 0,\ \tilde{x} = 0,\ \omega_s = 0\}$$

is UGAS.

*Proof:* The speed assignment is now satisfied as it is in $\mathcal{M}$. Let $Z := \left[z^\top,\tilde{x}^\top,\omega_s\right]^\top$. With the bounds

$$\begin{aligned} p_1|Z|^2 &\leq V \leq p_2|Z|^2 \\ \dot{V} &\leq -p_3|Z|^2 = -\alpha_3(|Z|) \end{aligned}$$

where $p_1 = \min(p_m,\ \frac{1}{2\mu\lambda},\ p_{o.m})$, $p_2 = \max(p_M,\ \frac{1}{2\mu\lambda},\ p_{o,M})$ and $p_3 = \min(q_m,\ \frac{1}{\mu},\ c_2)$, the Lyapunov function $V$ in (10) is bounded, and hence $Z$ is bounded on the maximal interval of existence. The assumed smoothness of plant dynamics and boundedness of all path and speed assignment signals implies that the

Fig. 1. Offshore supply vessel *Northern Clipper* of length $L = 76.2$ m.

right-hand side of (12) depends continuously on $(\theta, t)$ through bounded functions, and is bounded when $Z$ is bounded. This implies that there are no finite escape times and $|(z, \tilde{x}, \omega_s, \theta, t)|_{\mathcal{M}} = |Z|$. Furthermore, with (12) forward complete, $V$ radially unbounded, and $\alpha_3 \in \mathcal{K}$, $\mathcal{M}$ is UGAS and the maneuvering problem with output feedback is solved. ■

## III. Case: Output Feedback Maneuvering for a Marine Surface Vessel

An area where maneuvering design is of specific interest is in marine control systems. Applications include a single ship following a path (for instance in difficult maneuvering environments where it is important to move along a safe route), replenishment operations between several ships, docking operations, or seabed scanning. In this section, the output-feedback design of a maneuvering system will be applied for a ship interconnected with a observer.

Assume that only position measurement are available (e.g. GPS signals and gyro compass measurements) so an observer is needed to reconstruct the velocity states. In addition, the observer is used to filter out the oscillatory motion due to the waves so the control system only needs to counteract the slowly-varying disturbances.

Consider a vessel model for low-speed applications (up to 2-3 m/s) and station-keeping. For these cases the vessel can be described accurately with a linear model. Extension to maneuvering at higher speeds can be done by using a nonlinear model [12]. Let $\eta = [x, y, \psi]^\top$ be the earth fixed position vector, where $(x, y)$ is the position on the ocean surface and $\psi$ is the yaw angle (heading) and let $\nu = [u, v, r]^\top$ be the body-fixed velocity vector. The system model for a single ship with a linear wave frequency model $x_w \in \mathbb{R}^{12}$, bias $b \in \mathbb{R}^3$, and equations of motion in surge, sway, and yaw are written

$$\dot{x}_w = A_w x_w \quad (13a)$$

$$\dot{\eta} = R(\psi)\nu \quad (13b)$$

$$\dot{b} = -T_b^{-1} b \quad (13c)$$

$$M\dot{\nu} + D\nu = R(\psi)^\top b + u_c \quad (13d)$$

$$y = \eta + C_w x_w, \quad (13e)$$

where $A_w,\ C_w$ are matrices from a state-space representation of a linear wave spectra, $R(\psi) \in SO(3)$ is the rotation matrix, $T_b$ is used to describe the slowly-varying environmental disturbances, $M = M^\top > 0$ is the system inertia matrix including the hydrodynamic added inertia, $D$ is the hydrodynamic damping matrix, and $u_c$ is the fully actuated vector of control forces. For more details regarding ship modeling, the reader is suggested to consult [13] and [14]. Note that $R(\psi)^{-1} = R(\psi)^\top$. The desired path is given by $\xi(\theta) = [x_d(\theta),\ y_d(\theta),\ \psi_d(\theta)]^\top$ where $\theta$ is the parametrization variable and the desired heading is computed as

$$\psi_d(\theta) = \arctan\left(\frac{y_d^\theta(\theta)}{x_d^\theta(\theta)}\right). \quad (14)$$

When the path is parametrized in terms of path length, $\theta$ will have the unit 'meter', and the speed assignment $v_s(\theta, t)$ will then correspond to a desired tangential speed along the path. The following observer from [4], [6] is used:

$$\begin{aligned}
\dot{\hat{x}}_w &= A_w \hat{x}_w + K_1 \tilde{y} \\
\dot{\hat{\eta}} &= R(\psi)\hat{\nu} + K_2 \tilde{y} \\
\dot{\hat{b}} &= -T_b^{-1}\hat{b} + K_3 \tilde{y} \\
\dot{\hat{\nu}} &= -M^{-1}D\hat{\nu} + R(\psi)^\top \hat{b} + M^{-1}u_c + K_4 R^\top(\psi)\tilde{y} \\
\hat{y} &= \hat{\eta} + C_w \hat{x}_w, \text{ where } \tilde{y} = y - \hat{y}.
\end{aligned}$$

The model can be written as

$$\begin{aligned}
\dot{\eta} &= R(\psi)(\nu + \tilde{\nu}) \\
\dot{\hat{\nu}} &= -M^{-1}D\hat{\nu} + R(\psi)^\top \hat{b} + M^{-1}u_c + K_4 R^\top(\psi)\tilde{y}.
\end{aligned}$$

In the remaining $R = R(\psi)$ is used for simplicity. By choosing model matrices, $A_w, T_b$, and observer gains, $K_i,\ i = 1, ..., 4$, that commute with $R(\psi)$ [13, Prop. 6.1], the stability of the error dynamics can be shown to be independent of the rotation matrix. [6, Lemma 4.1]. Denote $\tilde{x} = [\tilde{x}_w^\top,\ \tilde{\eta}^\top,\ \tilde{b}^\top,\ \tilde{\nu}^\top]^\top$ and the observer error dynamics on state-space form are

$$\dot{\tilde{x}} = A_o(\psi)\tilde{x} = T(\psi)^\top A_o T(\psi)\tilde{x} \quad (15)$$

where

$$A_o = \begin{bmatrix} A_w - K_1 C_w & -K_1 & 0 & 0 \\ -K_2 C_w & -K_2 & 0 & I \\ -K_3 C_w & -K_3 & -T_b^{-1} & 0 \\ -K_4 C_w & -K_4 & 0 & -M^{-1}D \end{bmatrix}$$

and $T(\psi) = \operatorname{diag}(R(\psi)^\top,\ R(\psi)^\top,\ R(\psi)^\top,\ I)$ such that

$$T(\psi) A_o(\psi) T(\psi)^\top = A_o = \text{constant}. \quad (16)$$

If $A_o$ is Hurwitz, and if there exists a $P_o = P_o^\top > 0$ s.t. $P_o A_o + A_o^\top P_o \leq -I$, then by [6, Lemma 4.1], the equilibrium, $\tilde{x} = 0$, of the observer error dynamics is UGES.

### A. Output-feedback Control Design

Define the following error vectors

$$z_1(\eta,\theta) := \eta - \xi(\theta) \tag{17}$$
$$z_2(\eta,\hat{\nu},\theta,t) := \hat{\nu} - \alpha_1(\eta,\theta,t) \tag{18}$$
$$\omega_s(\dot{\theta},\theta,t) := \upsilon_s(\theta,t) - \dot{\theta}, \tag{19}$$

where $\alpha_1$ is the virtual control law to be defined in Step 1 below.

**Step 1:** Differentiate $z_1$ :

$$\begin{aligned}\dot{z}_1 &= \dot{\eta} - \xi^{\theta}(\theta)\dot{\theta} = R\hat{\nu} + R\tilde{\nu} - \xi^{\theta}\upsilon_s + \xi^{\theta}\omega_s \\ &= Rz_2 + R\alpha_1 + R\tilde{\nu} - \xi^{\theta}\upsilon_s + \xi^{\theta}\omega_s,\end{aligned}$$

and define the step 1 clf

$$V_1(\eta,\theta,\tilde{x}) = z_1^{\top}P_1z_1 + \frac{1}{d_1}\tilde{x}^{\top}P_o\tilde{x}, \quad P_i = P_i^{\top} > 0, \; i = 0,1.$$

The design procedure in Section II gives the following signals for the first step

$$\begin{aligned}\tau_1 &= \tau_1(\eta,\theta) = 2z_1^{\top}P_1\xi^{\theta} \\ \alpha_1(\eta,\theta,t) &= R^{\top}\left[A_1z_1 + \xi^{\theta}\upsilon_s + \alpha_0\right] \\ \alpha_0 &= -\kappa_1 P, \quad \kappa_1 > 0 \\ \dot{\alpha}_1 &= \sigma_1 + \alpha_1^{\theta}\dot{\theta}, \; \sigma_1 = \alpha_1^{\eta}\dot{\eta} + \alpha_1^{t},\end{aligned} \tag{20}$$

and the time-derivative of $V_1$ becomes

$$\dot{V}_1 \leq -z_1^{\top}Q_1z_1 + 2z_1^{\top}P_1Rz_2 + \tau_1\omega_s - c_1\tilde{x}^{\top}\tilde{x}$$

where $c_1 = \frac{1}{d_1} - \frac{1}{2\kappa_1} > 0$.

**Step 2:** The step 2 clf is defined as

$$V_2(x,\tilde{x},\theta,t) = V_1 + z_2^{\top}P_2z_2 + \frac{1}{d_2}\tilde{x}^{\top}P_o\tilde{x}, \quad P_2 = P_2^{\top} > 0.$$

Define control law $u_c$ and final tuning function $\tau_2$ as

$$\begin{aligned}&u_c = u_c(x,\tilde{x},\theta,t) = M[A_2z_2 - P_2^{-1}R^{\top}P_1z_1 \\ &+M^{-1}D\hat{\nu} - R^{\top}\hat{b} + \sigma_1 + \alpha_1^{\theta}\upsilon_s + u_0] \\ &u_0 = -\kappa_2K_4K_4P_2, \; \kappa_2 > 0 \\ &\tau_2(x,\hat{x},\theta) = \tau_1 + 2z_2^{\top}P_2\alpha_1^{\theta}.\end{aligned} \tag{21}$$

Denote $z := \left[z_1^{\top}, \; z_2^{\top}\right]^{\top}$, $P := \operatorname{diag}(P_1,P_2)$, and $Q := \operatorname{diag}(Q_1, \; Q_2)$. The time-derivative of $V_2$ is bounded by

$$\dot{V}_2 \leq -z^{\top}Q_z + \tau_2\omega_s - c_2\tilde{x}^{\top}\tilde{x}, \; c_2 = c_1 + \frac{1}{d_2} - \frac{1}{2\kappa_2} > 0.$$

The resulting $z$-dynamics is

$$\dot{z} = Fz + 2\mu gg^{\top}Pz + H(\tilde{x}),$$

where

$$\begin{aligned}F &= \begin{bmatrix} A_1 - \kappa_1P_1 & R \\ -P_2^{-1}R^{\top}P_1 & A_2 - \kappa_2K_4K_4P_2 \end{bmatrix}, \\ g &= \begin{bmatrix} \xi^{\theta} \\ \alpha_1^{\theta} \end{bmatrix}, \; H(\tilde{x}) = \begin{bmatrix} R\tilde{\nu} \\ K_4R^{\top}\tilde{y} \end{bmatrix}.\end{aligned}$$

Finally, the gradient update law $\omega_s = -\mu\tau_2$ renders $\dot{V}_2$ negative semidefinite

$$\dot{V}_2 \leq -z^{\top}Qz - \mu\tau_2^2 - c_2\tilde{x}^{\top}\tilde{x} \leq 0, \; \mu > 0.$$

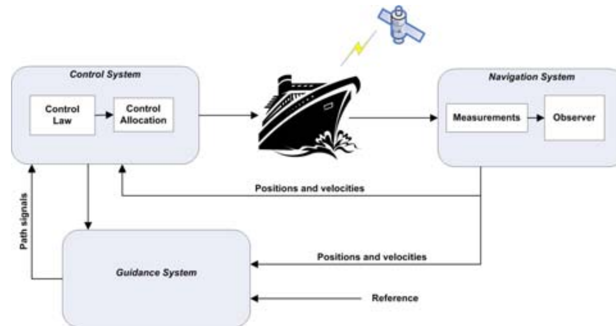


Fig. 2. An automatic control system for a ship.

*Theorem 3:* The closed-loop maneuvering system

$$\begin{aligned}\dot{z} &= Fz + 2\mu gg^{\top}Pz + H(\tilde{x}) \\ \dot{\theta} &= \upsilon_s(\theta,t) + 2\mu g^{\top}Pz \\ \dot{\tilde{x}} &= A_o\tilde{x}\end{aligned}$$

is forward complete and solves the maneuvering problem with output feedback problem , i.e. the set

$$\mathcal{M} = \{(z,\tilde{x},\theta,t) : z = 0, \; \tilde{x} = 0\}$$

is UGAS.

*Proof:* The proof of this theorem follows along the same lines as Theorem 1. ■

A comparison of the control laws, (20) and (21), with the control designs done for vessel in [10], [15], shows that the control laws are almost identical with the exception of the damping terms appearing in the output-feedback control laws. This motivates the belief that in order to guarantee output-feedback stability of these systems with, the state-feedback control design matrices $A_1$ and $A_2$ must be adjusted such that observer errors are dominated, and the inequalities, such as (4) and (6), hold.

This design can be seen as a fully automatic control system for a ship, see Figure 2, consisting of Guidance, Navigation and Control blocks. The navigation system uses the position measurements to estimate the unavailable states, and feed this to the guidance and the control systems. The control system consists of the control law (21) and the update law for $\theta$ with all necessary signals, while the guidance system incorporates the path $\xi(\theta)$, speed assignment $\upsilon_s(\theta,t)$, and their partial derivatives.

## IV. Simulation Results

To illustrate the suggested design procedure, a maneuvering operation along a circle shaped path is implemented with the proposed observer and output feedback controller. The control plant is exposed to waves and measurement noise. The path is the circle

$$\xi(\theta) = \begin{bmatrix} x_d(\theta) \\ y_d(\theta) \\ \psi_d(\theta) \end{bmatrix} = \begin{bmatrix} r\cos\left(\frac{\theta}{r}\right) \\ r\sin\left(\frac{\theta}{r}\right) \\ \arctan\left(\frac{x_d^{\theta}(\theta)}{y_d^{\theta}(\theta)}\right) \end{bmatrix}$$

where $r$, the radius of the circle, is 500 m. The desired surge speed starts at $\upsilon_s = 2$ m/s, and is set to $\upsilon_s = 3$

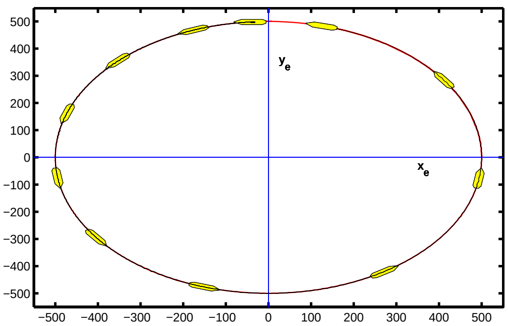


Fig. 3. Position plot of ship exposed to waves.

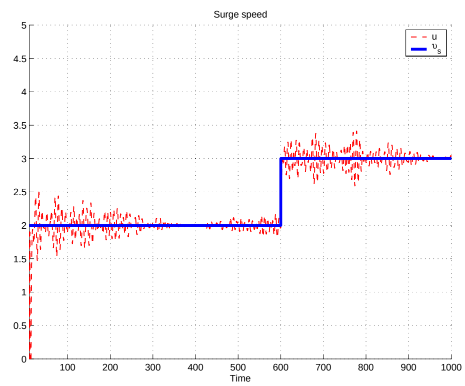


Fig. 4. Estimated surge speed $u$ and desired path speed $\upsilon_s$.

m/s after $t = 600$ s. The control plant model of the supply vessel "Northern Clipper" in Figure 1 consists of the nondimensional system matrices

$$M = \begin{bmatrix} 1.1274 & 0 & 0 \\ 0 & 1.8902 & -0.0744 \\ 0 & -0.0744 & 0.1278 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.0358 & 0 & 0 \\ 0 & 0.1183 & -0.0124 \\ 0 & -0.0041 & 0.0308 \end{bmatrix}$$

which are Bis-scaled coefficients, identified by full-scale sea trials in the North Sea [16]. The controller and observer parameters are set as $(I = I_{3\times 3})$ $A_1 = -0.5I$, $A_2 = -\operatorname{diag}(2,2,20)$, $P_1 = 0.6I$ $P_2 = \operatorname{diag}(10,10,40)$, $\kappa_1 = 5$, $\kappa_2 = 1$, $\mu = 10$ and $K_1 = [2.2I,\ I,\ I,\ I]$, $K_2 = 250I$, $K_3 = 100I$, $K_4 = 0.1I$. The initial conditions for the vessel are $\eta(0) = [0, 500, -\pi]$, $\nu(0) = [0,0,1]^\top$ and $\theta(0) = \pi/2$, and for the observer $\hat{\eta}(0) = [0, 500, -\pi]$, $\hat{\nu}(0) = [0,0,1]^\top$. The bias time constants are $T_b = 1000I$ and the wave model parameters are chosen corresponding to a wave period of 7.0 (s) in surge, sway, and yaw.

Figure 3 shows the output response of the vessels and shows how the vessel follows the path. Figure 4 show the commanded surge speed and the resulting estimated surge speed.

## V. Conclusion

An nonlinear output-feedback control design method is proposed for maneuvering systems. The method relies on the existence of an exponentially stable observer. Compared to state-feedback control laws, an additional damping term is added in the control law to ensure stability of the closed-loop system when using a combination of measured and estimated states in the control law. As a case study, a control law for maneuvering of a nonlinear vessel model has been designed, and simulated to demonstrate and validate the theoretical results.

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