The Marine Systems Simulator State-Space Model Representation for Dynamically Positioned Surface Vessels

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ABSTRACT: This paper presents a nonlinear state-space model suitable for numerical simulation and ship motion control system testing for dynamic positioning (DP) operations. This model represents a typical application of the tools included in the Marine Systems Simulator (MSS), and illustrates the modelling philosophy of the MSS. This simulator is a Matlab®/Simulink®-based toolbox specially developed at The Norwegian University of Science and Technology (NTNU) for education, rapid prototyping and evaluation of marine control systems. The model presented here is based on results of standard potential theory tools, which are used to obtain radiation and wave excitation forces. The fluid memory effects associated with the radiation forces are expressed as a reduced-order state-space model. This is a key issue, which allows one to replace the traditional low-/wave-frequency motion superposition principle by a unified state-space model in body-fixed coordinates. The resulting model is suitable for simulation of marine operations requiring station-keeping and low-speed ship manoeuvring in a seaway, and can incorporate viscous forces, wave drift, current loads, wind loads, multi-body interactions, and ship motion control systems.

1 INTRODUCTION

Numerical simulators have become an indispensable tool for control system design and analysis. In the case of ships performing marine operations, simulation scenarios can easily become complex when including models of the environment, vessels, mooring systems, risers, cranes with suspended loads, pipe-line equipment, ship motion control systems, *etc.*

At NTNU, there has been a continuous research focus on marine control system simulation tools during the past 10 years. Recently, the Marine Cybernetics Simulator (Sørensen *et al.*, 2003) and the Guidance Navigation and Control toolbox (Fossen, 2002) have been merged under the so-called Marine Systems Simulator (MSS, 2004). This simulator is as a Matlab®/ Simulink®-based toolbox specially developed for rapid prototyping and evaluation of marine control systems. MSS includes a set of Matlab functions and several Simulink libraries that cover a range of simulation scenarios based on model detail, user expertise and various applications of marine control systems for course-keeping, path-following, dynamic positioning, roll stabilization, *etc.*

This paper presents a particular application that illustrates the modeling philosophy of MSS: a nonlinear state-space model for dynamically positioned surface vessels. A key aspect of this approach is the state-space formulation of the equations of motion in terms of body-fixed coordinates adopted for MSS. Traditional models used for control system design and testing use data generated from standard hydrodynamic computer programs, which compute hydrodynamic coefficients and motion response transfer functions due to waves (RAO–response amplitude operators). These models together with data obtained from captive model testing in calm water (zero-frequency coefficients) provide one with the traditional framework that uses the superposition of the wave-frequency motion and the low-frequency motion. This approach is suitable for studying wave-vessel interactions for single vessels. In marine operations, however, vessels interact with other physical systems; thus, these types of operations requires models that includes energy exchange between the interacting systems. Motivated by the work of Bishop and Price (1981) and Bailey *et al.* (1998) on a unified model for manoeuvring in a seaway, an approach enabling this consists of using force transfer functions to represent the first-order wave excitation forces and express the fluid memory effects associated with the radiation forces in a state-space form. By doing so, other effects like viscous forces, wave drift, current, and wind loads, multi-body interactions, and ship motion control systems can be incorporated into the model by means of force superposition. This modular approach is the core philosophy of the MSS. To facilitate this, the model presented by Fossen and Smogeli (2004) is here extended to incorporate current, wind and second-order wave drift forces, as well as external forces from interacting systems. The coordinate transformations from the reference frames used in hydrodynamics to the body-fixed coordinates used for control and the conversion of the convolution integrals associated with potential damping terms into a state-space form are put into a rigid mathematical framework, and simulation results for a particular vessel are used to illustrate the use of the models in the MSS.

2 EQUATIONS OF MOTION

This section presents the 6 degrees-of-freedom (DOF) equations of motion using vectorial mechanics. Emphasis is placed on keeping the kinematics nonlinear while linear theory is assumed for the hydrodynamic forces and moments. In the following, a linear velocity vector in the point O decomposed in reference frame n is denoted as $\mathbf{v}_o^n$,and the angular velocity of frame b with respect to frame n decomposed in frame n and b are denoted $\boldsymbol{\omega}_{nb}^n$ and $\boldsymbol{\omega}_{nb}^b$ respectively.

2.1 *Coordinate Systems*

Three orthogonal coordinate systems are used to describe the vessel motions in 6DOF; see Figure 1.

*North-East-Down frame (*n*-frame)*: The n-frame $X_nY_nZ_n$ is assumed fixed on the Earth surface with the X_n-axis pointing North, the Y_n-axis pointing East, and the Z_n-axis down. It is considered inertial. The n-frame position $\mathbf{p}^n = [n, e, d]^\top$ (north, east, down) and Euler angles $\boldsymbol{\Theta} = [\phi, \theta, \psi]^\top$ (roll, pitch, yaw) are defined in terms of the generalized coordinate vector $\boldsymbol{\eta}$ (Fossen, 2002):

$$\boldsymbol{\eta} = [(\mathbf{p}^n)^\top, \boldsymbol{\Theta}^\top]^\top = [n, e, d, \phi, \theta, \psi]^\top. \quad (1)$$

*Hydrodynamic frame (*h*-frame)*: The hydrodynamic forces and moments are defined in a steadily translating hydrodynamic coordinate system $X_hY_hZ_h$ moving along the mean path of the ship with the constant speed U with respect to the n-frame. For DP, $U = 0$. The X_hY_h-plane is parallel to the mean water surface, the Z_h-axis is positive downwards, the Y_h-axis is positive towards starboard, and the X_h-axis is positive forwards, coinciding with the time-average yaw angle of the vessel $\bar{\psi}$. The h-frame is considered inertial, and the ship carries out oscillations about the steadily translating frame $X_hY_hZ_h$. The h-frame origin is denoted W while the h-frame generalized position vector (describing the incremental displacement and rotations of the vessel with respect to the h-frame) is:

$$\boldsymbol{\xi} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^\top. \quad (2)$$

Note that the incremental heave position $\xi_3 = d$. The incremental Euler angles $\boldsymbol{\Theta}^*$, which take the h-frame into the orientation of the b-frame, are defined in terms of the Euler angles ψ, θ, and ϕ according to:

$$\boldsymbol{\Theta}^* = \begin{bmatrix} \xi_4 \\ \xi_5 \\ \xi_6 \end{bmatrix} \triangleq \begin{bmatrix} \delta\phi \\ \delta\theta \\ \delta\psi \end{bmatrix} = \begin{bmatrix} \phi \\ \theta \\ \psi - \bar{\psi} \end{bmatrix}, \quad (3)$$

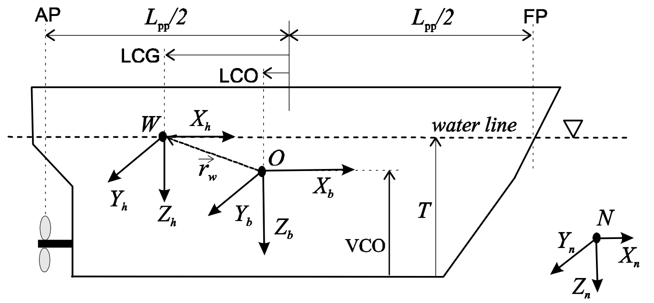

Figure 1: Definitions of coordinate systems with origins: W (h-frame), O (b-frame), and N (n-frame). T is the draught.

such that $\delta\psi$ represents the wave-frequency oscillations about a slowly-varying (constant for straight-line motion) yaw angle $\bar{\psi}$.

*Body-fixed frame (*b*-frame)*: The b-frame $X_bY_bZ_b$ is fixed to the hull. The coordinate origin is denoted O and is located on the center line a distance LCO relative to $L_{pp}/2$ (positive backwards) and a distance VCO relative to the baseline (positive upwards), where L_{pp} is the length between the perpendiculars. The center of gravity G with respect to O is located at $\mathbf{r}_g^b = [x_g, y_g, z_g]^\top$, while the h-frame origin W with respect to O is located at $\mathbf{r}_w^b = [x_w, y_w, z_w]^\top$. The X_b-axis is positive toward the bow, the Y_b-axis is positive towards starboard, and the Z_b-axis is positive downwards. Consequently, the body-fixed b-frame carries out oscillations about the steadily translating h-frame. The b-frame translational velocities $\mathbf{v}_o^b = [u, v, w]^\top$ (surge, sway, heave) in O and angular velocities of the b-frame with respect to the n-frame expressed in the b-frame $\boldsymbol{\omega}_{nb}^b = [p, q, r]^\top$ (roll, pitch, yaw) are defined in terms of the generalized velocity vector $\boldsymbol{\nu}$ (Fossen, 2002):

$$\boldsymbol{\nu} = [(\mathbf{v}_o^b)^\top, (\boldsymbol{\omega}_{nb}^b)^\top]^\top = [u, v, w, p, q, r]^\top. \quad (4)$$

Generally, the specific hydrodynamic program used to calculate the hydrodynamic properties of the vessel defines a local reference frame with axes different from the h-frame. Consequently, the data sets from this program must be transformed to the h-frame by using rotation matrices.

2.2 *Kinematics (*b*-frame to* n*-frame)*

From Fossen (2002), the translational velocity transformation between the b- and n-frame is:

$$\mathbf{v}_o^n = \mathbf{R}_b^n(\boldsymbol{\Theta})\mathbf{v}_o^b, \quad (5)$$

where $\mathbf{v}_o^n = \dot{\mathbf{p}}^n$, and the Euler angle rotation matrix (zyx-convention) between the n- and b-frame $\mathbf{R}_b^n(\boldsymbol{\Theta}) \in \mathbb{R}^{3\times3}$ is defined as:

$$\mathbf{R}_b^n(\boldsymbol{\Theta}) = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\phi s\theta s\psi & -c\psi s\phi + s\theta s\psi c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}. \quad (6)$$

Here, $\mathrm{s}\cdot = \sin(\cdot)$ and $\mathrm{c}\cdot = \cos(\cdot)$. Note that the rotation matrix $\mathbf{R}_a^b$ between any two frames a and b (from a to b) has the special properties that $(\mathbf{R}_a^b)^{-1} = (\mathbf{R}_a^b)^\top = \mathbf{R}_b^a$. The Euler angle rates satisfy:

$$\dot{\boldsymbol{\Theta}} = \mathbf{T}_\Theta(\boldsymbol{\Theta})\boldsymbol{\omega}_{bn}^b, \tag{7}$$

where $\mathbf{T}_\Theta(\boldsymbol{\Theta}) \in \mathbb{R}^{3\times3}$ is the Euler angle attitude transformation matrix:

$$\mathbf{T}_\Theta(\boldsymbol{\Theta}) = \begin{bmatrix} 1 & \mathrm{s}\phi\mathrm{t}\theta & \mathrm{c}\phi\mathrm{t}\theta \\ 0 & \mathrm{c}\phi & -\mathrm{s}\phi \\ 0 & \mathrm{s}\phi/\mathrm{c}\theta & \mathrm{c}\phi/\mathrm{c}\theta \end{bmatrix}, \quad \theta \neq \pm\frac{\pi}{2}, \tag{8}$$

and $\mathrm{t}\cdot = \tan(\cdot)$. Consequently:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\Theta})\boldsymbol{\nu}, \tag{9}$$

where $\mathbf{J}(\boldsymbol{\Theta}) \in \mathbb{R}^{6\times6}$ is the generalized velocity transformation matrix:

$$\mathbf{J}(\boldsymbol{\Theta}) = \begin{bmatrix} \mathbf{R}_b^n(\boldsymbol{\Theta}) & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{T}_\Theta(\boldsymbol{\Theta}) \end{bmatrix}, \quad \theta \neq \pm\frac{\pi}{2}. \tag{10}$$

2.3 *Kinematics (*b*-frame to* h*-frame)*

The transformation of a generalized velocity $\boldsymbol{\nu}$ in the b-frame to a generalized velocity $\dot{\boldsymbol{\xi}} = [(\mathbf{v}_w^h)^\top, (\dot{\boldsymbol{\Theta}}^*)^\top]^\top$ in the h-frame is done in two steps: a rotation and a translation. The translational velocity $\mathbf{v}_w^b$ of W and the angular velocity of the b-frame with respect to the h-frame $\boldsymbol{\omega}_{hb}^b$ decomposed in the b-frame are given by:

$$\mathbf{v}_w^b = \mathbf{v}_o^b + \boldsymbol{\omega}_{nb}^b \times \mathbf{r}_w^b, \tag{11}$$
$$\boldsymbol{\omega}_{hb}^b = \boldsymbol{\omega}_{nb}^b. \tag{12}$$

where $\mathbf{r}_w^b$ is the vector from O to the mean position of W. The angular velocity of the h-frame with respect to the n-frame $\boldsymbol{\omega}_{nh}^b$ is approximately zero, since the h-frame is considered inertial (only a slowly-varying yaw angle is assumed). The vector cross product $\times$ is defined in terms of the matrix $\mathbf{S}(\mathbf{r}_w^b) \in \mathbb{R}^{3\times3}$ such that:

$$\boldsymbol{\omega}_{nb}^b \times \mathbf{r}_w^b \triangleq -\mathbf{S}(\mathbf{r}_w^b)\boldsymbol{\omega}_{nb}^b = \mathbf{S}(\mathbf{r}_w^b)^\top\boldsymbol{\omega}_{nb}^b, \tag{13}$$

where:

$$\mathbf{S}(\mathbf{r}_w^b) = -\mathbf{S}^\top(\mathbf{r}_w^b) = \begin{bmatrix} 0 & -z_w & y_w \\ z_w & 0 & -x_w \\ -y_w & x_w & 0 \end{bmatrix}. \tag{14}$$

Introducing the *screw transformation* $\mathbf{H}(\mathbf{r}_w^b) \in \mathbb{R}^{6\times6}$:

$$\mathbf{H}(\mathbf{r}_w^b) \triangleq \begin{bmatrix} \mathbf{I}_{3\times3} & \mathbf{S}(\mathbf{r}_w^b)^\top \\ \mathbf{0}_{3\times3} & \mathbf{I}_{3\times3} \end{bmatrix}, \tag{15}$$

it is clear that:

$$\begin{bmatrix} \mathbf{v}_w^b \\ \boldsymbol{\omega}_{hb}^b \end{bmatrix} = \mathbf{H}(\mathbf{r}_w^b)\begin{bmatrix} \mathbf{v}_o^b \\ \boldsymbol{\omega}_{nb}^b \end{bmatrix}. \tag{16}$$

The rotation matrix from the b-frame to the h-frame is defined as $\mathbf{R}_b^h(\boldsymbol{\Theta}^*)$ such that:

$$\mathbf{v}_w^h = \mathbf{R}_b^h(\boldsymbol{\Theta}^*)\mathbf{v}_w^b. \tag{17}$$

Similarly, the angular velocity transformation from the b-frame to the h-frame becomes:

$$\dot{\boldsymbol{\Theta}}^* = \mathbf{T}_\Theta(\boldsymbol{\Theta}^*)\boldsymbol{\omega}_{hb}^b. \tag{18}$$

Note that $\dot{\boldsymbol{\Theta}}^* \neq \boldsymbol{\omega}_{hb}^h$, since $\boldsymbol{\omega}_{hb}^h = \mathbf{R}_b^h(\boldsymbol{\Theta}^*)\boldsymbol{\omega}_{hb}^b$. From (16), (17), and (18) it follows that:

$$\begin{aligned}\begin{bmatrix} \mathbf{v}_w^h \\ \dot{\boldsymbol{\Theta}}^* \end{bmatrix} &= \begin{bmatrix} \mathbf{R}_h^b(\boldsymbol{\Theta}^*) & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{T}_\Theta(\boldsymbol{\Theta}^*) \end{bmatrix}\begin{bmatrix} \mathbf{v}_w^b \\ \boldsymbol{\omega}_{hb}^b \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{R}_h^b(\boldsymbol{\Theta}^*) & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{T}_\Theta(\boldsymbol{\Theta}^*) \end{bmatrix}\mathbf{H}(\mathbf{r}_w^b)\begin{bmatrix} \mathbf{v}_o^b \\ \boldsymbol{\omega}_{nb}^b \end{bmatrix}.\end{aligned} \tag{19}$$

The generalized velocity transformation between the h and b frames then becomes:

$$\dot{\boldsymbol{\xi}} = \mathbf{J}^*(\boldsymbol{\Theta}^*)\boldsymbol{\nu}, \tag{20}$$

where $\mathbf{J}^*(\boldsymbol{\Theta}^*) \in \mathbb{R}^{6\times6}$ is a generalized velocity transformation matrix:

$$\mathbf{J}^*(\boldsymbol{\Theta}^*) = \begin{bmatrix} \mathbf{R}_h^b(\boldsymbol{\Theta}^*) & \mathbf{R}_h^b(\boldsymbol{\Theta}^*)\mathbf{S}(\mathbf{r}_w^b)^\top \\ \mathbf{0}_{3\times3} & \mathbf{T}_\Theta(\boldsymbol{\Theta}^*) \end{bmatrix}. \tag{21}$$

For most applications, the roll and pitch oscillations will be small, *i.e.* $\boldsymbol{\Theta}^* \approx \mathbf{0}$, such that:

$$\mathbf{R}_b^h(\boldsymbol{\Theta}^*) \approx \mathbf{I}_{3\times3}, \tag{22a}$$
$$\mathbf{T}_\Theta(\boldsymbol{\Theta}^*) \approx \mathbf{I}_{3\times3}, \tag{22b}$$
$$\mathbf{J}^* \triangleq \mathbf{J}^*(\mathbf{0}) = \mathbf{H}(\mathbf{r}_w^b), \tag{22c}$$

where now $\mathbf{J}^*$ is a constant matrix.

Assumption A1 The oscillations $\boldsymbol{\Theta}^*$ of the b-frame with respect to the h-frame are small such that (22) holds.

Note that assumption A1 only applies to the transformation of forces and matrices between the b-frame and the h-frame, while the nonlinear kinematics between the n-frame and the b-frame (9) are preserved and hence valid for large Euler angles $\boldsymbol{\Theta}$.

2.4 *Kinetics*

The generalized forces acting on the vessel are found by formulating Newton's 2nd law in b-frame coordinates. Since the hydrodynamic forces and moments are computed in h-frame coordinates these will be transformed to the b-frame. Note that using (21) and assumption A1, a generalized force $\boldsymbol{\tau}$ (containing forces and moments) in the b-frame can be transformed to the generalized force $\boldsymbol{\tau}^*$ in the h-frame by:

$$\boldsymbol{\tau}^* = (\mathbf{J}^*)^{-\top}\boldsymbol{\tau}. \tag{23}$$

The 6DOF rigid-body equations of motion in the b-frame are (Fossen, 2002):

$$\mathbf{M}_{RB}\dot{\boldsymbol{\nu}} + \mathbf{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}_H + \boldsymbol{\tau}, \qquad (24)$$

where $\boldsymbol{\tau}_H \in \mathbb{R}^6$ contains generalized hydrodynamic forces, $\boldsymbol{\tau} \in \mathbb{R}^6$ is a generalized vector containing environmental, propulsion and external forces, $\mathbf{C}_{RB} \in \mathbb{R}^{6\times6}$ is the rigid-body Coriolis and centripetal matrix, and $\mathbf{M}_{RB} \in \mathbb{R}^{6\times6}$ is the rigid-body system inertia matrix:

$$\mathbf{M}_{RB} = \left[\begin{array}{cc} m\mathbf{I}_{3\times3} & -m\mathbf{S}(\mathbf{r}_g^b) \\ m\mathbf{S}(\mathbf{r}_g^b) & \mathbf{I}_o \end{array} \right], \qquad (25)$$

where m is the vessel mass, $\mathbf{r}_g^b$ is the vector from the center of gravity to the body frame origin, and $\mathbf{I}_o = \mathbf{I}_o^\top > 0$ is the inertia tensor. The generalized force vectors and inertia matrix are computed with respect to the coordinate origin O.

Assumption A2 The Coriolis and centripetal forces due to a rotating b-frame and added mass are neglected in the following.

Assumption A2 is justifiable in low-speed applications like DP, where these terms are dominated by damping and feedback. However, they may be easily included in the final model if desired. Under assumption A1 and A2 (*i.e.* $\mathbf{C}_{RB} = \mathbf{0}_{6\times6}$ and $\boldsymbol{\Theta}^*$ small), the b-frame equations of motion can be transformed to the h-frame by using (20):

$$\ddot{\boldsymbol{\xi}} = \mathbf{J}^*\dot{\boldsymbol{\nu}}, \qquad (26)$$

Pre-multiplying by $(\mathbf{J}^*)^{-\top}$, (24) can be written:

$$(\mathbf{J}^*)^{-\top}\mathbf{M}_{RB}(\mathbf{J}^*)^{-1}\ddot{\boldsymbol{\xi}} = (\mathbf{J}^*)^{-\top}(\boldsymbol{\tau}_H + \boldsymbol{\tau}). \qquad (27)$$

The generalized forces in the h-frame are defined as $\boldsymbol{\tau}_H^* = (\mathbf{J}^*)^{-\top}\boldsymbol{\tau}_H$ and $\boldsymbol{\tau}^* = (\mathbf{J}^*)^{-\top}\boldsymbol{\tau}$ such that:

$$\mathbf{M}_{RB}^*\ddot{\boldsymbol{\xi}} = \boldsymbol{\tau}_H^* + \boldsymbol{\tau}^*, \qquad (28)$$

where $\mathbf{M}_{RB}^* \triangleq (\mathbf{J}^*)^{-\top}\mathbf{M}_{RB}(\mathbf{J}^*)^{-1}$.

3 HYDRODYNAMIC FORCES

The generalized hydrodynamic forces in the h-frame can be expanded according to Faltinsen (1990):

$$\boldsymbol{\tau}_H^* = \boldsymbol{\tau}_{HS}^* + \boldsymbol{\tau}_R^* + \boldsymbol{\tau}_{W1}^*, \qquad (29)$$

where $\boldsymbol{\tau}_{HS}^*$ is the generalized *hydrostatic* force, $\boldsymbol{\tau}_R^*$ is the generalized *radiation* force representing fluid memory effects, and $\boldsymbol{\tau}_{W1}^*$ is the generalized *first-order wave excitation* force, consisting of Froude-Krylov and diffraction forces. The generalized hydrostatic forces, contributing only in heave, roll, and pitch, are written as:

$$\boldsymbol{\tau}_{HS}^* = -\mathbf{g}^*(\boldsymbol{\xi}).$$

In the traditional hydrodynamic models it is assumed that the response of the vessel is linear. Thus superposition can be applied, and the hydrodynamic forces can be computed in the frequency domain. Then, for sinusoidal excitation and motion, the generalized radiation forces can be written as

$$\boldsymbol{\tau}_R^* = -\mathbf{A}^*(\omega)\ddot{\boldsymbol{\xi}} - \mathbf{B}^*(\omega)\dot{\boldsymbol{\xi}}, \qquad (30)$$

where $\mathbf{A}^*(\omega) \in \mathbb{R}^{6\times6}$ is the frequency dependent added mass and $\mathbf{B}^*(\omega) \in \mathbb{R}^{6\times6}$ is the frequency dependent potential damping. Note that under assumption A2, $\mathbf{B}^*(\omega)$ is defined for zero forward speed such that the Coriolis and centripetal terms due to added mass are zero and $\mathbf{B} = \mathbf{B}^\top$. Inserting (30) in the equations of motion (28), the h-frame representation becomes:

$$(\mathbf{M}_{RB}^* + \mathbf{A}^*(\omega))\ddot{\boldsymbol{\xi}} + \mathbf{B}^*(\omega)\dot{\boldsymbol{\xi}} + \mathbf{g}^*(\boldsymbol{\xi}) = \boldsymbol{\tau}_W^* + \boldsymbol{\tau}^*. \qquad (31)$$

Hydrodynamic programs like WAMIT (2004), ShipX (VERES) (Fathi, 2004), SEAWAY (Journée and Adegeest, 2003), *etc.*, can be used to compute $\mathbf{A}^*(\omega)$ and $\mathbf{B}^*(\omega)$ in terms of coefficient tables, and the generalized first-order wave excitation force $\boldsymbol{\tau}_{W1}^*$ in terms of force transfer functions. These terms are all computed in the h-frame. The radiation-induced forces and moments $\boldsymbol{\tau}_R^*$ are functions of frequency and time. In the next section another form of the equations of motion, valid for more general excitation forces, is presented.

3.1 *Time-Domain Representation*

From Cummins (1962) and Ogilvie (1964), the frequency dependent terms $\mathbf{A}^*(\omega)$ and $\mathbf{B}^*(\omega)$ can be removed from (31) by writing the equations of motion in the following form:

$$\begin{array}{c}(\mathbf{M}_{RB}^* + \mathbf{A}^*(\infty))\ddot{\boldsymbol{\xi}} + \mathbf{B}^*(\infty)\dot{\boldsymbol{\xi}} + \boldsymbol{\mu}^* \\ +\mathbf{g}^*(\boldsymbol{\xi}) = \boldsymbol{\tau}_{W1}^* + \boldsymbol{\tau}^*,\end{array} \qquad (32)$$

where $\mathbf{A}^*(\infty) = \mathbf{A}^*(\infty)^\top \in \mathbb{R}^{6\times6}$ and $\mathbf{B}^*(\infty)$ are constant matrices evaluated at the infinite frequency, and $\boldsymbol{\mu}^*$ is a potential damping term defined as:

$$\boldsymbol{\mu}^* \triangleq \int_{-\infty}^{t} \mathbf{K}^*(t-\tau)\dot{\boldsymbol{\xi}}(t)d\tau. \qquad (33)$$

$\mathbf{K}^*(\tau) \in \mathbb{R}^{6\times6}$ is a matrix of *retardation functions*, and may be expressed as:

$$\mathbf{K}^*(\tau) = \frac{2}{\pi}\int_0^\infty (\mathbf{B}^*(\omega) - \mathbf{B}^*(\infty))\cos(\omega\tau)d\omega. \quad (34)$$

Note that $\mathbf{K}^*(\tau)$ can be computed off-line using the $\mathbf{B}^*(\omega)$ data set. For details, including how to add viscous damping and additional numerical considerations, consult Fossen and Smogeli (2004).

Kristiansen and Egeland (2003) have proposed a state-space formulation for the frequency dependent damping term in (32). Since for causal systems $\mathbf{K}^*(t-\tau) = \mathbf{0}$ for $t < 0$, (33) is rewritten as:

$$\boldsymbol{\mu}^*(t) \overset{\text{causal}}{=} \int_0^t \mathbf{K}^*(t-\tau)\dot{\boldsymbol{\xi}}(t)d\tau. \qquad (35)$$

If $\dot{\boldsymbol{\xi}}$ is a unit impulse, then $\boldsymbol{\mu}^*(t)$ given by (35) will be an *impulse response* function. Consequently, $\boldsymbol{\mu}^*(t)$ can be approximated by a linear state-space model:

$$\begin{aligned} \dot{\boldsymbol{\chi}} &= \mathbf{A}_r\boldsymbol{\chi} + \mathbf{B}_r\dot{\boldsymbol{\xi}}, \quad \boldsymbol{\chi}(0) = \mathbf{0}, \\ \boldsymbol{\mu}^* &= \mathbf{C}_r\boldsymbol{\chi} + \mathbf{D}_r\dot{\boldsymbol{\xi}}, \qquad (36) \end{aligned}$$

where $(\mathbf{A}_r, \mathbf{B}_r, \mathbf{C}_r, \mathbf{D}_r)$ are constant matrices of appropriate dimensions. Transforming (32) to the b-frame, the equations of motion becomes:

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{D}\boldsymbol{\nu} + \boldsymbol{\mu} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}_{W1} + \boldsymbol{\tau}, \qquad (37)$$

where the following transformations based on assumption A1 are used:

$$\begin{aligned} \mathbf{M} &= \mathbf{M}_{RB} + \mathbf{J}^{*\top}\mathbf{A}^*(\infty)\mathbf{J}^*, && (38a) \\ \mathbf{D} &= \mathbf{J}^{*\top}\mathbf{B}^*(\infty)\mathbf{J}^*, && (38b) \\ \boldsymbol{\mu} &= \mathbf{J}^{*\top}\boldsymbol{\mu}^*, && (38c) \\ \mathbf{g}(\boldsymbol{\eta}) &= \mathbf{J}^{*\top}\mathbf{g}^*(\boldsymbol{\xi}), && (38d) \\ \boldsymbol{\tau}_{W1} + \boldsymbol{\tau} &= \mathbf{J}^{*\top}(\boldsymbol{\tau}^*_{W1} + \boldsymbol{\tau}^*), && (38e) \end{aligned}$$

Notice that the properties $\mathbf{M} = \mathbf{M}^\top > 0$ and $\dot{\mathbf{M}} = \mathbf{0}$ also holds for this model since the generalized added mass matrix $\mathbf{A}^*(\infty)$ is constant and symmetric. The resulting nonlinear state-space model from (9), (37), and (36) is:

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \mathbf{J}(\boldsymbol{\Theta})\boldsymbol{\nu}, && (39a) \\ \mathbf{M}\dot{\boldsymbol{\nu}} &= -\mathbf{D}\boldsymbol{\nu} - \mathbf{J}^{*\top}\underbrace{(\mathbf{C}_r\boldsymbol{\chi} + \mathbf{D}_r\mathbf{J}^*\boldsymbol{\nu})}_{\boldsymbol{\mu}^*} \\ &\quad - \mathbf{g}(\boldsymbol{\eta}) + \boldsymbol{\tau}_{W1} + \boldsymbol{\tau}, && (39b) \\ \dot{\boldsymbol{\chi}} &= \mathbf{A}_r\boldsymbol{\chi} + \mathbf{B}_r\underbrace{\mathbf{J}^*\boldsymbol{\nu}}_{\dot{\boldsymbol{\xi}}}, \qquad \boldsymbol{\chi}(0) = \mathbf{0}. && (39c) \end{aligned}$$

4 ENVIRONMENTAL MODELS

4.1 *Waves*

Irregular waves are commonly described by a wave spectrum $S(\omega,\beta) = S(\omega)D(\beta)$, where the frequency spectrum $S(\omega)$ describes the energy distribution of the sea state over different frequencies ω, and the spreading function $D(\beta)$ describes the distribution of wave energy over directions β in the n-frame. Common frequency spectra and spreading functions may be found in *e.g.* Ochi (1998). For simulation, the sea state is realized as a superposition of harmonic components extracted from the wave spectrum, where the harmonic component j is defined in terms of the amplitude ζ_j, frequency ω_j, direction β_j, and random phase ϕ_j.

4.2 *Wind*

The wind is commonly parameterized by the velocity $U_w(z,t) = \bar{U}_w(z) + U_g(t)$ and the direction β_w in the n-frame, where $\bar{U}_w(z)$ is a height dependent mean value and $U_g(t)$ is a fluctuating gust component defined from a wind gust spectrum. Commonly used gust spectra are the Harris wind spectrum (Det Norske Veritas, 2000) and the NORSOK spectrum (NORSOK, 1999).

4.3 *Current*

For surface vessels, only the surface current is of any importance. If the current is given by magnitude U_c and direction β_c in the n-frame, the current velocity vector relative to the vessel $\mathbf{v}_c^b$ may be written as

$$\mathbf{v}_c^b = U_c[\cos(\beta_c - \psi), \sin(\beta_c - \psi), 0]^\intercal. \qquad (40)$$

5 LOAD MODELS

This section details the implementation of the first-order wave loads $\boldsymbol{\tau}_{W1}$, and introduces additional environmental loads $\boldsymbol{\tau}_E$ from current, wind, and second-order wave drift forces. How to include external forces from interacting systems $\boldsymbol{\tau}_I$ (mooring systems, risers etc.) is also considered. Propulsion forces $\boldsymbol{\tau}_P$ are not discussed further, since accurate propulsion models are outside the scope of this paper. The generalized force vector $\boldsymbol{\tau}$ in (24) is therefore rewritten according to:

$$\boldsymbol{\tau} = \boldsymbol{\tau}_E + \boldsymbol{\tau}_P + \boldsymbol{\tau}_I. \qquad (41)$$

5.1 *Transformation tools*

The inclusion of the forces $\boldsymbol{\tau}$ in the b-frame equations of motion (39) typically requires transformations of positions, velocities, accelerations, and forces from one coordinate to another. Following the approach in Section 2.3, the position $\mathbf{x}_p^n$, velocity $\mathbf{v}_p^b$ and acceleration $\dot{\mathbf{v}}_p^b$ of a point p located at $\mathbf{r}_p^b$ in the b-frame can be expressed as:

$$\begin{aligned} \mathbf{x}_p^n &= \boldsymbol{\eta} + \mathbf{R}_b^n(\boldsymbol{\Theta})\mathbf{r}_p^b, && (42) \\ \mathbf{v}_p^b &= \mathbf{H}(\mathbf{r}_p^b)\boldsymbol{\nu}, && (43) \\ \dot{\mathbf{v}}_p^b &= \mathbf{H}(\mathbf{r}_p^b)\dot{\boldsymbol{\nu}}, && (44) \end{aligned}$$

where $\mathbf{R}_b^n(\boldsymbol{\Theta})$ and $\mathbf{H}(\mathbf{r}_p^b)$ are defined by (6) and (15) respectively. Correspondingly, the generalized force vector $\boldsymbol{\tau}_p$ with point of attack p is transformed to the origin O by:

$$\boldsymbol{\tau} = \mathbf{H}^\intercal(\mathbf{r}_p^b)\boldsymbol{\tau}_p. \qquad (45)$$

For details, see Fossen (2002). These tools can be used as input to an interacting system, *e.g.* to find the velocity and accelerations of the tip of a crane. The force generated by an interacting system, *e.g.* the force on the crane tip from a suspended load, can then be transformed back to the origin of the b-frame, and be included in the equations of motion.

5.2 *First-order wave forces*

The first-order wave excitation forces in the h-frame $\boldsymbol{\tau}^*_{W1} = [\tau^{*1}_{W1}, \cdots, \tau^{*6}_{W1}]$ are calculated from the force transfer functions $T^i(\omega_j, \beta_j)$ (for DOF i), which are outputs from a hydrodynamic analysis program and tabulated as functions of the wave frequency ω_j and the wave heading relative to the vessel $\gamma_{rj} = \beta_j - \psi$. The total excitation force is the sum over N harmonic wave components, i.e. for $i = 1..6$:

$$\tau^{*i}_{W1} = \sum_{j=1}^{N} \zeta_j \left|T^i_j\right| \cos(\omega_j t + \phi_j + \arg(T^i_j)). \quad (46)$$

5.3 *Wave drift forces*

The wave drift loads are a significant part of the total excitation forces, and may divided into a mean and a slowly varying component. For long-crested waves, the determination of the drift forces can be done by means of a set of approximations given in Newman (1974) and Faltinsen (1990). A hydrodynamic program like WAMIT (2004) can determine the quadratic transfer functions T^{i2}_{jj} (for wave component j and DOF i) from first-order potential theory, with T^{i2}_{jj} tabulated as functions of ω_j and γ_{rj}. For $i = 1..6$, the total wave drift force τ^i_{W2} is then given by:

$$\tau^i_{W2} = \sum_{j=1}^{N}\sum_{k=1}^{N} \zeta_j \zeta_k T^{i2}_{jk} \cos\left((\omega_k - \omega_j)t + \varphi_k - \varphi_j\right), \quad (47)$$

where φ_j is a phase angle and $T^{i2}_{jk} = T^{i2}_{kj} = \frac{1}{2}\left(T^{i2}_{jj} + T^{i2}_{kk}\right)$. Note that the mean wave drift force given by $j = k$ is valid for both long- and short-crested waves, whereas the slowly-varying drift force for $j \neq k$ only is valid for long-crested waves. The resulting wave drift load vector in the b-frame $\boldsymbol{\tau}_{W2}$ is found from (45) using the vector from O to the point of attack of the wave drift forces $\mathbf{r}^b_{W2}$:

$$\boldsymbol{\tau}_{W2} = \mathbf{H}^\intercal(\mathbf{r}^b_{W2})[\ \tau^1_{W2}, \ \cdots \ , \tau^6_{W2}\ ]^\intercal. \quad (48)$$

5.4 *Nonlinear damping/current forces*

The linear drag component from the current is included in the equations of motion (39) by replacing the vessel velocity $\boldsymbol{\nu}$ in the damping term $\mathbf{D}\boldsymbol{\nu}$ with the relative velocity $\boldsymbol{\nu}_r$:

$$\boldsymbol{\nu}_r = \boldsymbol{\nu} - \mathbf{v}^b_c. \quad (49)$$

Quadratic damping is not as easily included, but several options exist. In surge, the viscous damping f_{c1} may be modelled by *e.g.* (Lewis, 1989):

$$f_{c1} = \frac{\rho_w}{2} S u_r^2 (1+k)(C_f + \Delta C_f), \quad (50)$$

$$C_f = \frac{0.075}{(\log_{10} R_n - 2)^2}, \quad (51)$$

where ρ_w is the density of water, S is the wetted surface of the hull, $u_r = u - U_c\cos(\beta_c - \psi)$ is the relative velocity in surge, k is the form factor giving a viscous correction, C_f is the flat plate friction from the ITTC 1957 line, ΔC_f represents friction due to hull roughness, and R_n is the Reynolds number. For relative current angles $\gamma_{cr} = |\beta_c - \psi| \gg 0$ the cross flow principle (Faltinsen, 1990) may be applied to calculate the nonlinear damping force in sway f_{c2} and moment in yaw f_{c6}:

$$f_{c2} = \frac{\rho_w}{2}\int_{L_{pp}} T(x) C_D(x) v^x_r(x)|v^x_r(x)|dx, \quad (52)$$

$$f_{c6} = \frac{\rho_w}{2}\int_{L_{pp}} xT(x) C_D(x) v^x_r(x)|v^x_r(x)|dx. \quad (53)$$

Here, $T(x)$ is the draft, $C_D(x)$ is the 2 dimensional drag coefficient, and $v^x_r(x) = v_r + rx$ is the relative cross-flow velocity at x. $v_r = v - U_c \sin(\beta_c - \psi)$ is the relative velocity in sway, and r is the angular velocity in yaw. Drag coefficients for different hull forms may be found in *e.g.* Hooft (1994). From (45) the resulting nonlinear damping term in the b-frame $\mathbf{d}(\boldsymbol{\nu}_r)$ becomes:

$$\mathbf{d}(\boldsymbol{\nu}_r) = -\mathbf{H}^\intercal(\mathbf{r}^b_c)[\ f_{c1} \quad f_{c2} \quad 0 \quad 0 \quad 0 \quad f_{c6}\ ]^\intercal, \quad (54)$$

where $\mathbf{r}^b_c$ is the vector from O to the point of attack of the drag forces c. A typical value could be $\mathbf{r}^b_c = [0, 0, T/2]^\intercal$. If available, quadratic drag coefficients from experiments or a CFD analysis may replace the ITTC drag and cross-flow formulations. The relative generalized velocity $\boldsymbol{\nu}_r$ should replace $\boldsymbol{\nu}$ in all hydrodynamic computations.

5.5 *Wind forces*

The wind angle relative to the vessel is defined as $\gamma_{wr} = \beta_w - \psi$. The wind drag coefficients C_{w1}, C_{w2}, and C_{w6} in surge, sway, and yaw are tabulated as functions of γ_{wr}, giving the wind forces f_{w1}, f_{w2}, and f_{w6}:

$$f_{wi} = 0.5 \rho_a C_{wi}(\gamma_{wr}) A_{wi} U_w |U_w|, \quad (55)$$

where $i \in \{1, 2, 6\}$, ρ_a is the density of air, A_{w1} and A_{w2} are the transverse and lateral projected areas of the hull above the waterline and the superstructure, and $A_{w6} = A_{w2}L_{oa}$, where L_{oa} is the overall length of the vessel. Wind drag coefficient tables may be found in *e.g.* Blendermann (1986). The resulting wind load vector in the b-frame $\boldsymbol{\tau}_{wind}$ is found in a similar manner as in (54) using the vector from O to the point of attack of the wind forces $\mathbf{r}^b_{wind}$.

6 THE RESULTING MODEL

Combining the nonlinear state-space model (39) and the load models from Section 5, the resulting

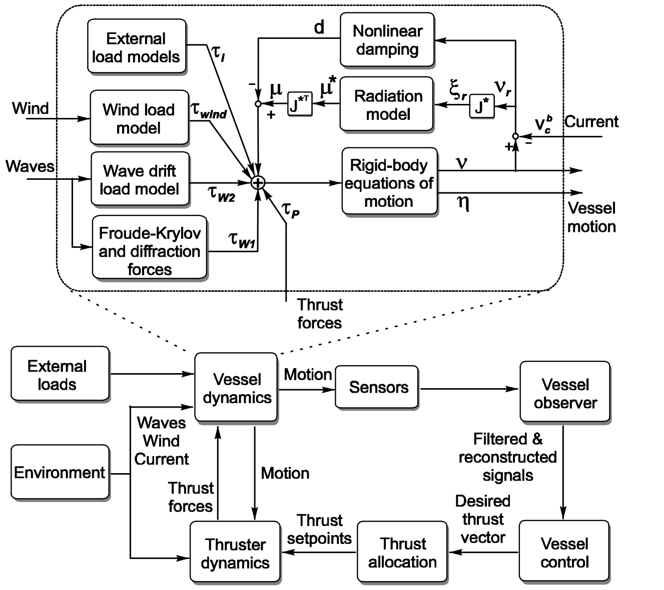

Figure 2: Block diagram of a DP operation as modelled in MSS, with focus on the vessel dynamics.

vessel model implemented in the MSS is:

$$\begin{aligned} \dot{\boldsymbol{\eta}} &= \mathbf{J}(\boldsymbol{\Theta})\boldsymbol{\nu}, \\ \mathbf{M}\dot{\boldsymbol{\nu}} &= -\mathbf{D}\boldsymbol{\nu}_r - \mathbf{d}(\boldsymbol{\nu}_r) - \mathbf{J}^{*\top}(\mathbf{C}_r\boldsymbol{\chi} + \mathbf{D}_r\mathbf{J}^*\boldsymbol{\nu}_r) \\ &\quad - \mathbf{g}(\boldsymbol{\eta}) + \boldsymbol{\tau}_{W1} + \boldsymbol{\tau}_{W2} + \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_P + \boldsymbol{\tau}_I, \\ \dot{\boldsymbol{\chi}} &= \mathbf{A}_r\boldsymbol{\chi} + \mathbf{B}_r\mathbf{J}^*\boldsymbol{\nu}_r, \qquad \boldsymbol{\chi}(0) = \mathbf{0}. \end{aligned} \tag{56}$$

Figure 2 shows a block diagram of a DP operation as given by (56), focusing on the vessel dynamics with the rigid-body equations of motion, and forces from nonlinear damping, radiation, first-order wave excitation, wave drift, wind, propulsion and external loads.

7 SIMULATION RESULTS

Simulations were performed with the S-175 container ship, with main particulars given in Table 1. ShipX (VERES) (Fathi, 2004) was used to compute the frequency dependent added mass and damping, as well as the first-order exciting wave force transfer functions. For details on the frequency dependent coefficients and computation of the retardation functions, consult Fossen and Smogeli (2004). The simulation shows a DP operation with mean wave direction 45 degrees off the bow. The sea state was generated from the JONSWAP wave spectrum with significant wave height $H_s = 4$ m, wave peak frequency $\omega_p = 0.60$ rad/s, spectrum peakedness factor $\gamma = 3.3$, and a directional spreading function giving short-crested waves. At time $t = 100$ s a crane load of 1000 kN is applied at the point $\mathbf{r}_p^b = [0, 15, 0]^\top$. Figure 3 shows time series of the 6DOF generalized first-order exciting wave forces $\boldsymbol{\tau}_{W1}$ and the radiation forces $\boldsymbol{\tau}_R$, and Figure 4 shows time series of the generalized position vector $\boldsymbol{\eta}$. Even though the crane load is applied as a step, the force superposition principle leads to physically natural vessel motions: the main effect is a new mean roll angle.

Table 1: The S-175 container ship main particulars.

Length between perpendiculars L_{pp}	175 m
Beam B	25.4 m
Draught T	9.5 m
Displaced volume ∇	24140 m^3
Block coefficient C_B	0.572
LCG relative to midships	-2.48 m

8 CONCLUSIONS

A nonlinear state-space model for surface vessels performing station-keeping and low-speed manoeuvring in a seaway has been presented. The model representation using force superposition rather than motion superposition was a key aspect, since it allowed a modular description of complex marine operations. This enabled the interaction of various physical systems like vessels, cranes, risers, and motion control systems. The presented models have been implemented in the Marine Systems Simulator (MSS) developed at NTNU.

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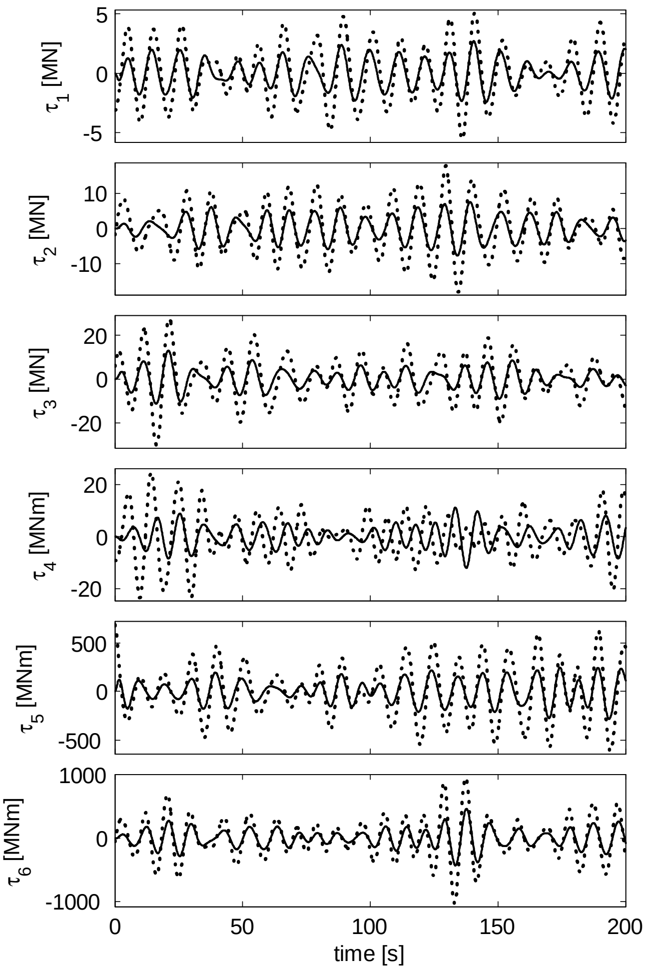

Figure 3: Time series of the 6DOF generalized first-order wave exciting forces $\boldsymbol{\tau}_{W1}$ (dotted) and radiation forces $\boldsymbol{\tau}_R$ (solid).

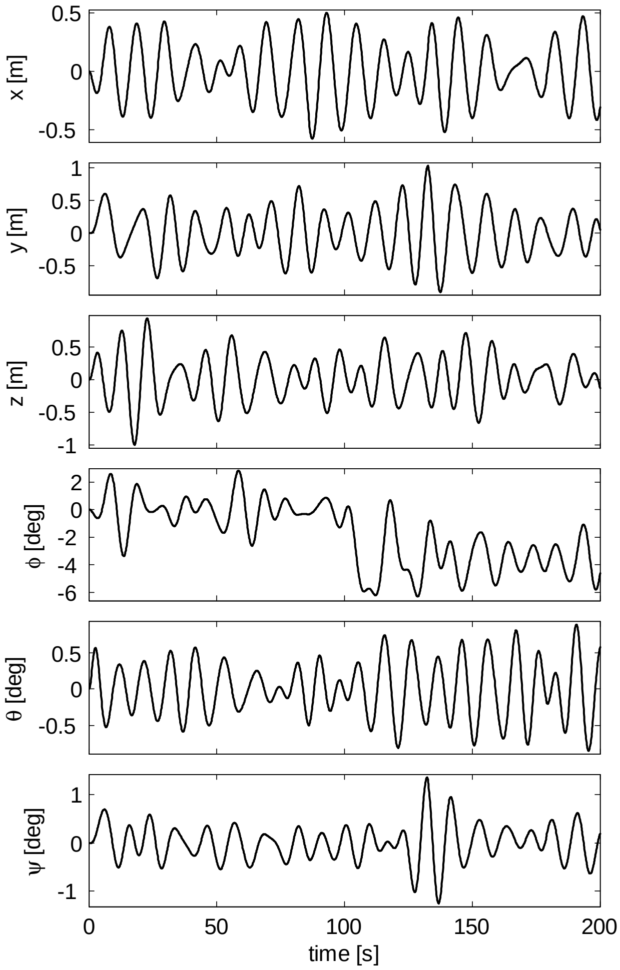

Figure 4: Time series of the generalized position vector $\boldsymbol{\eta} = [x, y, z, \phi, \theta, \psi]^\intercal$.

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