GUIDED FORMATION CONTROL FOR FULLY ACTUATED MARINE SURFACE CRAFT

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Abstract: This paper addresses the problem of formation control for fully actuated marine surface craft. Within a leader-follower framework, a guided formation control concept is developed by means of a three-step, backstepping-inspired, cascaded-based design procedure. Key qualities of the scheme encompass helmsman-like transient motion behavior and extendability toward underactuated vehicles.

Keywords: Guided formation control, Leader-follower framework, Fully actuated marine surface craft, Nonlinear model-based control, Guidance and synchronization law design

1. INTRODUCTION

The subject of formation control has been important at sea for centuries. In old times, groups of warships had to be controlled during naval battles. In both world wars, it was pivotal for merchant ships to travel in convoys. Today, applications include underway ship replenishment, towing of large structures at sea, and surveying of hydrocarbons. In the future, more sophisticated concepts will emerge, facilitated by new sensor, communication, and computer technology. Formations will become increasingly autonomous, consisting of fully autonomous marine craft that must operate in so-called dirty, dull, and dangerous environments. The vessels can act as scouts, nodes in communication and sensor networks, or elements within battlegroups. There is generally strength in numbers, and multi-vehicle operations render possible tasks that no single vehicle can solve, as well as increase operational robustness toward individual failures.

During recent years, the marine control community has focused considerably on formation control concepts. Most of the work seems to have been performed within a leader-follower framework, e.g., in (Encarnação and Pascoal 2001), where an autonomous underwater vehicle (AUV) tracks the planar projection of a surface craft onto its nominal path, while the surface craft follows its own path at sea; in (Skjetne *et al.* 2002), where formation control of multiple so-called maneuvering systems yields a robust scheme with dynamic adjustment to the weakest link in the formation; in (Lapierre *et al.* 2003), where coordination of two AUVs is achieved by augmenting a path parameter synchronization algorithm to the controller of the follower vehicle; in (Aguiar *et al.* 2006), where multiple AUVs are coordinated along spatial paths despite communication constraints; and in (Børhaug *et al.* 2006), where formation control of AUVs moving along straight lines are considered. Work related to virtual structures is reported in (Fiorelli *et al.* 2004), where cooperative control of AUVs is achieved through the use of virtual leaders and artificial potentials, considering the formation as a rigid-body geometric structure; and in (Ihle *et al.* 2005), where the approach is rooted in analytical mechanics for multi-body dynamics, facilitating a flexible and robust formation control scheme where the geometric constraints of a virtual structure are enforced by feedback control. Also, research in the vein of behavioral methods can be found in (Arrichiello *et al.* 2006), where marine surface vessels move in formation while avoiding collisions with environmental obstacles. Finally, the anthologies (Kumar *et al.* 2005) and (Pettersen *et al.* 2006) report state-of-the-art concepts for a broad number of formation control scenarios.

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The main contribution of this paper is a concept denoted *guided formation control*. Developed within a leader-follower framework, the scheme is based on principles from integrator backstepping design (Krstić *et al.* 1995) as well as theory for nonlinear time-varying cascades (Panteley *et al.* 1998). A key assumption is that the formation control designer is free to decide the path that the formation shall traverse.

Notation: The time derivative of (a vector) $\mathbf{x}(t)$ is denoted $\dot{\mathbf{x}}$, the partial derivative of $\mathbf{x}(\varpi(t))$ is denoted $\mathbf{x}'$ $\left(= \frac{\partial \mathbf{x}}{\partial \varpi}(\varpi(t))\right)$, while $|\cdot|$ represents the Euclidean vector norm as well as the induced matrix norm.

2. GUIDED FORMATION CONTROL

This section develops the concept of guided formation control for fully actuated marine surface craft.

2.1 Dynamic Model of a Marine Surface Craft

A 3 degree-of-freedom (DOF) dynamic model of the surge, sway, and yaw modes can be found in (Fossen 2002), and consists of the kinematics

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu}, \tag{1}$$

and the kinetics

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau} + \mathbf{R}(\psi)^\top \mathbf{b}, \tag{2}$$

where $\boldsymbol{\eta} = [x, y, \psi]^\top \in \mathbb{R}^2 \times \mathcal{S}$ represents the earth-fixed position and heading (with $\mathcal{S} = [-\pi, \pi]$), $\boldsymbol{\nu} = [u, v, r]^\top \in \mathbb{R}^3$ represents the vessel-fixed velocity, $\mathbf{R}(\psi) \in SO(3)$ is the transformation matrix

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

that transforms from the vessel-fixed BODY frame (**B**) to the earth-fixed NED frame (**N**), $\mathbf{M}$ is the inertia matrix, $\mathbf{C}(\boldsymbol{\nu})$ is the centrifugal and coriolis matrix, while $\mathbf{D}(\boldsymbol{\nu})$ is the hydrodynamic damping matrix. The system matrices satisfy the properties $\mathbf{M} = \mathbf{M}^\top > 0$, $\mathbf{C} = -\mathbf{C}^\top$ and $\mathbf{D} > 0$. The vessel-fixed propulsion forces and moment is represented by $\boldsymbol{\tau} = [\tau_\mathrm{X}, \tau_\mathrm{Y}, \tau_\mathrm{N}]^\top \in \mathbb{R}^3$, corresponding to a fully actuated vessel. Full actuation means that all 3 DOFs can be independently controlled simultaneously, i.e., the direction of the linear velocity is independent of the heading of the vessel. This is not the case for an underactuated craft, where the orientation of the linear velocity is inherently coupled (in a sense locked) to the heading. Finally, $\mathbf{b}$ represents the low-frequency earth-fixed environmental forces that act on the vessel.

2.2 Formation Control Scenario

This paper considers formation control within a leader-follower framework, where a formation structure is defined relative to a virtual formation leader, which constitutes the formation center (formation reference point). It is assumed that the formation structure, the path to be traversed by the leader, as well as the temporal motion of the leader, can all be chosen by the formation control designer. Consequently, consider a path continuously parameterized by a scalar variable $\varpi \in \mathbb{R}$, such that the position of a point belonging to the path is represented by $\mathbf{p}_\mathrm{p}(\varpi) \in \mathbb{R}^2$. Thus, the path is a one-dimensional manifold that can be expressed by the set

$$\mathcal{P} = \left\{ \mathbf{p} \in \mathbb{R}^2 \mid \mathbf{p} = \mathbf{p}_\mathrm{p}(\varpi)\ \forall \varpi \in \mathbb{R} \right\}. \tag{4}$$

Then, represent the virtual formation leader by $\mathbf{p}_\mathrm{l}(t) = \mathbf{p}_\mathrm{p}(\varpi_\mathrm{l}(t))$. The leader traverses the path by adhering to the speed profile $U_\mathrm{l}(\varpi_\mathrm{l})$, implemented through

$$\dot{\varpi}_\mathrm{l} = \frac{U_\mathrm{l}(\varpi_\mathrm{l})}{\left|\mathbf{p}'_\mathrm{p}(\varpi_\mathrm{l})\right|}, \tag{5}$$

since $|\dot{\mathbf{p}}_\mathrm{l}| = \left|\mathbf{p}'_\mathrm{p}(\varpi_\mathrm{l})\right| \dot{\varpi}_\mathrm{l} = U_\mathrm{l}(\varpi_\mathrm{l})$, where $U_\mathrm{l}(\varpi_\mathrm{l}) \in [U_{\mathrm{l,min}}, U_{\mathrm{l,max}}]$, $U_{\mathrm{l,min}} > 0$. Furthermore, consider a formation consisting of n members, each uniquely identified through the index set $\mathcal{I} = \{1, ..., n\}$. The assigned formation position for member i can be represented by $\mathbf{p}_{\mathrm{f,i}}(t)$, which is related to the formation leader through a chosen geometric assignment. It is assumed that $\mathbf{p}_{\mathrm{f,i}} \neq \mathbf{p}_{\mathrm{f,j}}\ \forall i \neq j$, where $i, j \in \mathcal{I}$.

2.2.1. Problem Statement In our scenario, the formation control problem for fully actuated marine surface craft can be stated by

$$\lim_{t\to\infty} \left(\boldsymbol{\eta}_\mathrm{i}(t) - \boldsymbol{\eta}_{\mathrm{f,i}}(t)\right) = \mathbf{0}\ \forall i \in \mathcal{I}, \tag{6}$$

where $\boldsymbol{\eta}_\mathrm{i}(t)$ represents the ith formation member, and $\boldsymbol{\eta}_{\mathrm{f,i}}(t) = \left[\mathbf{p}_{\mathrm{f,i}}^\top(t), \psi_{\mathrm{f,i}}(t)\right]^\top$ where $\psi_{\mathrm{f,i}}(t)$ can be any arbitrary heading (e.g., an auxiliary task objective).

2.3 Motion Control of Individual Formation Members

This section develops the control, guidance and synchronization laws that each formation member must employ in order to converge to its assigned position in the formation. The underlying motion control concept entails a three-step, backstepping-inspired, cascaded-based design procedure, and goes by the name of *guided motion control* (Breivik and Fossen 2006).

2.3.1. Step 1: Control Loop Design Since the position of a vessel can be controlled through its linear velocity, we redefine the output space from the nominal 3 DOF position and heading to the 3 DOF linear velocity and heading. Consequently, consider the positive definite and radially unbounded Control Lyapunov Function (CLF)

$$V_\mathrm{g} = \frac{1}{2}(z_\psi^2 + \mathbf{z}_\nu^\top \mathbf{M} \mathbf{z}_\nu + \tilde{\mathbf{b}}^\top \boldsymbol{\Gamma}^{-1} \tilde{\mathbf{b}}) \tag{7}$$

where we have

$$z_\psi = \psi - \psi_\mathrm{d} \tag{8}$$

and

$$\mathbf{z}_\nu = \boldsymbol{\nu} - \boldsymbol{\alpha}, \tag{9}$$

where $\boldsymbol{\alpha} = [\alpha_u, \alpha_v, \alpha_r]^\top \in \mathbb{R}^3$ is a so-called vector of stabilizing functions (virtual inputs that become reference signals) yet to be designed. Also,

$$\tilde{\mathbf{b}} = \hat{\mathbf{b}} - \mathbf{b} \tag{10}$$

represents an adaptation error where $\hat{\mathbf{b}}$ is the estimate of $\mathbf{b}$, and by assumption $\dot{\mathbf{b}} = \mathbf{0}$. Finally, $\boldsymbol{\Gamma} = \boldsymbol{\Gamma}^\top > 0$ is the so-called adaptation gain matrix.

Then, differentiate the CLF with respect to time to obtain

$$\begin{aligned}\dot{V}_{\mathrm{g}} &= z_\psi \dot{z}_\psi + \mathbf{z}_\nu^\top \mathbf{M}\dot{\mathbf{z}}_\nu + \tilde{\mathbf{b}}^\top \boldsymbol{\Gamma}^{-1}\dot{\tilde{\mathbf{b}}} \\ &= z_\psi(\dot{\psi} - \dot{\psi}_{\mathrm{d}}) + \mathbf{z}_\nu^\top \mathbf{M}(\dot{\boldsymbol{\nu}} - \dot{\boldsymbol{\alpha}}) + \tilde{\mathbf{b}}^\top \boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{b}}},\end{aligned}$$

which is equal to

$$\dot{V}_{\mathrm{g}} = z_\psi(\mathbf{h}^\top \dot{\boldsymbol{\eta}} - \dot{\psi}_{\mathrm{d}}) + \mathbf{z}_\nu^\top(\mathbf{M}\dot{\boldsymbol{\nu}} - \mathbf{M}\dot{\boldsymbol{\alpha}}) + \tilde{\mathbf{b}}^\top \boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{b}}}$$

by introducing

$$\mathbf{h} = [0, 0, 1]^\top . \tag{11}$$

Subsequently, recognizing that $\mathbf{h}^\top \dot{\boldsymbol{\eta}} = \mathbf{h}^\top \mathbf{R}\boldsymbol{\nu} = \mathbf{h}^\top \boldsymbol{\nu}$ and $\boldsymbol{\nu} = \mathbf{z}_\nu + \boldsymbol{\alpha}$, we obtain

$$\begin{aligned}\dot{V}_{\mathrm{g}} = {} & z_\psi(\mathbf{h}^\top \boldsymbol{\alpha} - \dot{\psi}_{\mathrm{d}}) + \mathbf{z}_\nu^\top(\boldsymbol{\tau} - \mathbf{C}\boldsymbol{\nu} - \mathbf{D}\boldsymbol{\nu} - \mathbf{M}\dot{\boldsymbol{\alpha}}) + \\ & \mathbf{z}_\nu^\top(\mathbf{R}^\top \mathbf{b} + \mathbf{h}z_\psi) + \tilde{\mathbf{b}}^\top \boldsymbol{\Gamma}^{-1}\dot{\hat{\mathbf{b}}},\end{aligned}$$

which results in

$$\begin{aligned}\dot{V}_{\mathrm{g}} = {} & -k_\psi z_\psi^2 - \mathbf{z}_\nu^\top \mathbf{C}\mathbf{z}_\nu - \mathbf{z}_\nu^\top \mathbf{D}\mathbf{z}_\nu + \\ & \mathbf{z}_\nu^\top(\boldsymbol{\tau} - \mathbf{C}\boldsymbol{\alpha} - \mathbf{D}\boldsymbol{\alpha} - \mathbf{M}\dot{\boldsymbol{\alpha}}) + \\ & \mathbf{z}_\nu^\top(\mathbf{R}^\top \hat{\mathbf{b}} + \mathbf{h}z_\psi) + \tilde{\mathbf{b}}^\top \boldsymbol{\Gamma}^{-1}(\dot{\hat{\mathbf{b}}} - \boldsymbol{\Gamma}\mathbf{R}\mathbf{z}_\nu)\end{aligned}$$

since $\mathbf{b} = \hat{\mathbf{b}} - \tilde{\mathbf{b}}$, and when choosing the virtual input $\mathbf{h}^\top \boldsymbol{\alpha} = \alpha_{\mathrm{r}}$ as

$$\alpha_{\mathrm{r}} = \dot{\psi}_{\mathrm{d}} - k_\psi z_\psi, \tag{12}$$

where $k_\psi > 0$ is a constant. Since $\mathbf{z}_\nu^\top \mathbf{C}(\boldsymbol{\nu})\mathbf{z}_\nu = 0$, by selecting the control input

$$\boldsymbol{\tau} = \mathbf{M}\dot{\boldsymbol{\alpha}} + \mathbf{C}\boldsymbol{\alpha} + \mathbf{D}\boldsymbol{\alpha} - \mathbf{R}^\top \hat{\mathbf{b}} - \mathbf{h}z_\psi - \mathbf{K}_\nu \mathbf{z}_\nu \tag{13}$$

where $\mathbf{K}_\nu = \mathbf{K}_\nu^\top > 0$ is a constant matrix, and by choosing the disturbance adaptation update law

$$\dot{\hat{\mathbf{b}}} = \boldsymbol{\Gamma}\mathbf{R}\mathbf{z}_\nu, \tag{14}$$

we finally obtain the negative semi-definite

$$\dot{V}_{\mathrm{g}} = -k_\psi z_\psi^2 - \mathbf{z}_\nu^\top(\mathbf{D} + \mathbf{K}_\nu)\mathbf{z}_\nu. \tag{15}$$

Considering the state vector $\mathbf{z}_{\mathrm{g}} = \left[z_\psi, \mathbf{z}_\nu^\top, \tilde{\mathbf{b}}^\top\right]^\top$, the following proposition can now be stated

Proposition 1. The equilibrium point $\mathbf{z}_{\mathrm{g}} = \mathbf{0}$ is rendered uniformly globally asymptotically and locally exponentially stable (UGAS/ULES) by adhering to (12), (13) and (14) under the assumption that $\boldsymbol{\alpha}$ and $\dot{\boldsymbol{\alpha}}$ are uniformly bounded.

PROOF. The proposed result follows by straightforward application of Theorem 1 in (Fossen *et al.* 2001).

Note that the developed controller cannot achieve anything meaningful unless it is fed sensible reference signals, i.e., unless $\boldsymbol{\alpha}_{\mathrm{v}} = [\alpha_{\mathrm{u}}, \alpha_{\mathrm{v}}]^\top \in \mathbb{R}^2$ is purposefully defined. This task is the responsibility of the final two design steps.

2.3.2. Step 2: Guidance Loop Design We now design the required *orientation* of $\boldsymbol{\alpha}_{\mathrm{v}}$ such that a marine surface craft controlled by (13) and (14) attains its assigned formation position relative to the path. Consequently, consider the positive definite and radially unbounded CLF

$$V_{\tilde{\varepsilon}} = \frac{1}{2}\tilde{\boldsymbol{\varepsilon}}^\top \tilde{\boldsymbol{\varepsilon}}, \tag{16}$$

with

$$\tilde{\boldsymbol{\varepsilon}} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{\mathrm{f}} \tag{17}$$

and

$$\boldsymbol{\varepsilon} = \mathbf{R}_{\mathrm{C}}^\top(\mathbf{p} - \mathbf{p}_{\mathrm{c}}), \tag{18}$$

where $\mathbf{p}_{\mathrm{c}} = \mathbf{p}_{\mathrm{p}}(\varpi_{\mathrm{c}})$ represents a *collaborator* point that acts cooperatively with the vessel as an intermediate path attractor, and whose intended purpose is to ensure that the surface craft can converge to its assigned formation position relative to the path (represented by $\boldsymbol{\varepsilon}_{\mathrm{f}}$), irrespective of whether it has synchronized with the formation leader or not. For a given ϖ_{c}, define a path-tangential reference frame at $\mathbf{p}_{\mathrm{c}}$ termed the COLLABORATOR frame (**C**). To arrive at **C**, the INERTIAL frame (**I**) must be positively rotated an angle

$$\chi_{\mathrm{c}} = \arctan\left(\frac{y_{\mathrm{p}}'(\varpi_{\mathrm{c}})}{x_{\mathrm{p}}'(\varpi_{\mathrm{c}})}\right), \tag{19}$$

which can be represented by the rotation matrix

$$\mathbf{R}_{\mathrm{C}} = \begin{bmatrix} \cos\chi_{\mathrm{c}} & -\sin\chi_{\mathrm{c}} \\ \sin\chi_{\mathrm{c}} & \cos\chi_{\mathrm{c}} \end{bmatrix}, \tag{20}$$

$\mathbf{R}_{\mathrm{C}} \in SO(2)$. Hence, equation (18) represents the error vector between the craft and its collaborator decomposed in **C**. The local coordinates $\boldsymbol{\varepsilon} = [s, e]^\top$ consist of the along-track error s and the cross-track error e. Thus, $\boldsymbol{\varepsilon}_{\mathrm{f}}$ represents the assigned path-relative formation position, and it is assumed that $\boldsymbol{\varepsilon}_{\mathrm{f}}$, $\dot{\boldsymbol{\varepsilon}}_{\mathrm{f}}$ and $\ddot{\boldsymbol{\varepsilon}}_{\mathrm{f}}$ are uniformly bounded. Typically, $\dot{\boldsymbol{\varepsilon}}_{\mathrm{f}} = \ddot{\boldsymbol{\varepsilon}}_{\mathrm{f}} = \mathbf{0}$.

Now, differentiate the CLF in (16) along the trajectories of $\tilde{\boldsymbol{\varepsilon}}$ to obtain

$$\begin{aligned}\dot{V}_{\tilde{\varepsilon}} &= \tilde{\boldsymbol{\varepsilon}}^\top \dot{\tilde{\boldsymbol{\varepsilon}}} \\ &= \tilde{\boldsymbol{\varepsilon}}^\top(\mathbf{S}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{C}}^\top(\mathbf{p} - \mathbf{p}_{\mathrm{c}}) + \mathbf{R}_{\mathrm{C}}^\top(\dot{\mathbf{p}} - \dot{\mathbf{p}}_{\mathrm{c}}) - \dot{\boldsymbol{\varepsilon}}_{\mathrm{f}}) \\ &= \tilde{\boldsymbol{\varepsilon}}^\top\left(\mathbf{S}_{\mathrm{C}}^\top \boldsymbol{\varepsilon} + \mathbf{R}_{\mathrm{C}}^\top \mathbf{H}\dot{\boldsymbol{\eta}} - \mathbf{v}_{\mathrm{c}} - \dot{\boldsymbol{\varepsilon}}_{\mathrm{f}}\right) \\ &= \tilde{\boldsymbol{\varepsilon}}^\top\left(\mathbf{S}_{\mathrm{C}}^\top \tilde{\boldsymbol{\varepsilon}} + \mathbf{S}_{\mathrm{C}}^\top \boldsymbol{\varepsilon}_{\mathrm{f}} + \mathbf{R}_{\mathrm{C}}^\top \mathbf{H}\dot{\boldsymbol{\eta}} - \mathbf{v}_{\mathrm{c}} - \dot{\boldsymbol{\varepsilon}}_{\mathrm{f}}\right) \\ &= \tilde{\boldsymbol{\varepsilon}}^\top\left(\mathbf{R}_{\mathrm{C}}^\top \mathbf{H}\mathbf{R}\boldsymbol{\nu} - \mathbf{v}_{\mathrm{c}} + \mathbf{S}_{\mathrm{C}}^\top \boldsymbol{\varepsilon}_{\mathrm{f}} - \dot{\boldsymbol{\varepsilon}}_{\mathrm{f}}\right)\end{aligned}$$

since $\dot{\mathbf{R}}_{\mathrm{C}} = \mathbf{R}_{\mathrm{C}}\mathbf{S}_{\mathrm{C}}$ with $\mathbf{S}_{\mathrm{C}} = -\mathbf{S}_{\mathrm{C}}^\top \Rightarrow \tilde{\boldsymbol{\varepsilon}}^\top \mathbf{S}_{\mathrm{C}}^\top \tilde{\boldsymbol{\varepsilon}} = 0$, and where

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \tag{21}$$

Also, $\mathbf{v}_{\mathrm{c}} = [U_{\mathrm{c}}, 0]^\top$, where $U_{\mathrm{c}} = |\dot{\mathbf{p}}_{\mathrm{c}}|$, represents the linear velocity of the collaborator decomposed in $\mathbf{C}$. Furthermore, we have that

$$\begin{aligned}\dot{V}_{\tilde{\varepsilon}} &= \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{C}}^\top \mathbf{H}\mathbf{R}\boldsymbol{\alpha} - \mathbf{v}_{\mathrm{c}} + \mathbf{S}_{\mathrm{C}}^\top \varepsilon_{\mathrm{f}} - \dot{\varepsilon}_{\mathrm{f}}\right) + \\ &\quad \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{H}\mathbf{R}\mathbf{z}_\nu \\ &= \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{C}}^\top \mathbf{H}\mathbf{R}\mathbf{H}^\top\mathbf{H}\boldsymbol{\alpha} - \mathbf{v}_{\mathrm{c}} + \mathbf{S}_{\mathrm{C}}^\top \varepsilon_{\mathrm{f}} - \dot{\varepsilon}_{\mathrm{f}}\right) + \\ &\quad \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{H}\mathbf{R}\mathbf{H}^\top\mathbf{H}\mathbf{z}_\nu,\end{aligned}$$

since $\boldsymbol{\nu} = \mathbf{z}_\nu + \boldsymbol{\alpha}$ and $\mathbf{H}^\top\mathbf{H} = \mathbf{I}_{3\times 3}$ (identity matrix). Defining $\mathbf{H}\mathbf{R}\mathbf{H}^\top = \mathbf{R}_{\mathrm{B}}$, i.e.,

$$\mathbf{R}_{\mathrm{B}} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}, \tag{22}$$

leads to

$$\begin{aligned}\dot{V}_{\tilde{\varepsilon}} &= \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\boldsymbol{\alpha} - \mathbf{v}_{\mathrm{c}} + \mathbf{S}_{\mathrm{C}}^\top \varepsilon_{\mathrm{f}} - \dot{\varepsilon}_{\mathrm{f}}\right) + \\ &\quad \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu,\end{aligned}$$

where $\mathbf{R}_{\mathrm{B}}\mathbf{H}\boldsymbol{\alpha} = \mathbf{R}_{\mathrm{B}}\boldsymbol{\alpha}_{\mathrm{v,B}}$ with $\boldsymbol{\alpha}_{\mathrm{v,B}} = \boldsymbol{\alpha}_{\mathrm{v}} = [\alpha_{\mathrm{u}}, \alpha_{\mathrm{v}}]^\top$. Now, $\mathbf{R}_{\mathrm{B}}\boldsymbol{\alpha}_{\mathrm{v,B}} = \boldsymbol{\alpha}_{\mathrm{v,I}} = \mathbf{R}_{\mathrm{DV}}\boldsymbol{\alpha}_{\mathrm{v,DV}}$ represents the desired linear velocity decomposed in $\mathbf{I}$, while $\boldsymbol{\alpha}_{\mathrm{v,DV}} = [U_{\mathrm{d}}, 0]^\top$, where $U_{\mathrm{d}} = |\boldsymbol{\alpha}_{\mathrm{v}}|$, represents the desired linear velocity decomposed in a DESIRED VELOCITY frame (**DV**). Hence, we get

$$\begin{aligned}\dot{V}_{\tilde{\varepsilon}} &= \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{C}}^\top \boldsymbol{\alpha}_{\mathrm{v,I}} - \mathbf{v}_{\mathrm{c}} + \mathbf{S}_{\mathrm{C}}^\top \varepsilon_{\mathrm{f}} - \dot{\varepsilon}_{\mathrm{f}}\right) + \\ &\quad \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu,\end{aligned} \tag{23}$$

which is equal to

$$\begin{aligned}\dot{V}_{\tilde{\varepsilon}} &= \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{DV}}\boldsymbol{\alpha}_{\mathrm{v,DV}} - \mathbf{v}_{\mathrm{c}} + \mathbf{S}_{\mathrm{C}}^\top \varepsilon_{\mathrm{f}} - \dot{\varepsilon}_{\mathrm{f}}\right) + \\ &\quad \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu \\ &= \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{R}}\boldsymbol{\alpha}_{\mathrm{v,DV}} - \mathbf{v}_{\mathrm{c}} + \mathbf{S}_{\mathrm{C}}^\top \varepsilon_{\mathrm{f}} - \dot{\varepsilon}_{\mathrm{f}}\right) + \\ &\quad \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu,\end{aligned}$$

where $\mathbf{R}_{\mathrm{C}}^\top(\chi_{\mathrm{c}})\mathbf{R}_{\mathrm{DV}}(\chi_{\mathrm{d}}) = \mathbf{R}_{\mathrm{R}}(\chi_{\mathrm{d}} - \chi_{\mathrm{c}})$, i.e., the relative orientation between **C** and **DV**.

Thus, U_{c} and $(\chi_{\mathrm{d}} - \chi_{\mathrm{c}})$ can be considered as virtual inputs for driving $\tilde{\varepsilon}$ to zero, given that $U_{\mathrm{d}} > 0$. Denote the angular difference by $\chi_{\mathrm{r}} = \chi_{\mathrm{d}} - \chi_{\mathrm{c}}$, and expand the CLF derivative to obtain

$$\begin{aligned}\dot{V}_{\tilde{\varepsilon}} &= \tilde{s}(U_{\mathrm{d}}\cos\chi_{\mathrm{r}} - U_{\mathrm{c}} + \dot{\chi}_{\mathrm{c}} e_{\mathrm{f}} - \dot{s}_{\mathrm{f}}) + \\ &\quad \tilde{e}(U_{\mathrm{d}}\sin\chi_{\mathrm{r}} - \dot{\chi}_{\mathrm{c}} s_{\mathrm{f}} - \dot{e}_{\mathrm{f}}) + \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu,\end{aligned}$$

where it becomes apparent that U_{c} and χ_{r} currently cannot be used to drive $\tilde{\varepsilon}$ to zero, not even if $\dot{\varepsilon}_{\mathrm{f}} = \mathbf{0}$.

Consequently, we introduce a *mediator* point, which is related to the real vessel such that when the vessel converges to its assigned formation position relative to the path, the mediator converges to the path. Hence, the relationship between the surface craft, the mediator, and the collaborator can be expressed by

$$\begin{aligned}\varepsilon &= \varepsilon_{\mathrm{m}} + \varepsilon_{\mathrm{f}} & (24)\\ &= \mathbf{R}_{\mathrm{C}}^\top(\mathbf{p}_{\mathrm{m}} - \mathbf{p}_{\mathrm{c}}) + \varepsilon_{\mathrm{f}} & (25)\\ &= \mathbf{R}_{\mathrm{C}}^\top(\mathbf{p} - \mathbf{p}_{\mathrm{c}}) & (26)\end{aligned}$$

such that

$$\mathbf{p}_{\mathrm{m}} = \mathbf{p} - \mathbf{R}_{\mathrm{C}}\varepsilon_{\mathrm{f}}, \tag{27}$$

which is used to compute the location of the mediator. This means that

$$\begin{aligned}\dot{\mathbf{p}}_{\mathrm{m}} &= \dot{\mathbf{p}} - \dot{\mathbf{R}}_{\mathrm{C}}\varepsilon_{\mathrm{f}} - \mathbf{R}_{\mathrm{C}}\dot{\varepsilon}_{\mathrm{f}} \\ &= \dot{\mathbf{p}} - \mathbf{R}_{\mathrm{C}}\mathbf{S}_{\mathrm{C}}\varepsilon_{\mathrm{f}} - \mathbf{R}_{\mathrm{C}}\dot{\varepsilon}_{\mathrm{f}}\end{aligned} \tag{28}$$

which shows that

$$\boldsymbol{\alpha}_{\mathrm{v,m,I}} = \boldsymbol{\alpha}_{\mathrm{v,I}} - \mathbf{R}_{\mathrm{C}}\mathbf{S}_{\mathrm{C}}\varepsilon_{\mathrm{f}} - \mathbf{R}_{\mathrm{C}}\dot{\varepsilon}_{\mathrm{f}}, \tag{29}$$

or rather, that

$$\boldsymbol{\alpha}_{\mathrm{v,I}} = \boldsymbol{\alpha}_{\mathrm{v,m,I}} + \mathbf{R}_{\mathrm{C}}(\mathbf{S}_{\mathrm{C}}\varepsilon_{\mathrm{f}} + \dot{\varepsilon}_{\mathrm{f}}). \tag{30}$$

Now, since $\varepsilon_{\mathrm{m}} = \tilde{\varepsilon}$ we duly insert (30) into (23) to get

$$\dot{V}_{\tilde{\varepsilon}} = \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{C}}^\top \boldsymbol{\alpha}_{\mathrm{v,m,I}} - \mathbf{v}_{\mathrm{c}}\right) + \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu$$

since $\mathbf{S}_{\mathrm{C}} = -\mathbf{S}_{\mathrm{C}}^\top$, which is equal to

$$\begin{aligned}\dot{V}_{\tilde{\varepsilon}} &= \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{DV}}\boldsymbol{\alpha}_{\mathrm{v,m,DV}} - \mathbf{v}_{\mathrm{c}}\right) + \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu \\ &= \tilde{\varepsilon}^\top \left(\mathbf{R}_{\mathrm{R}}\boldsymbol{\alpha}_{\mathrm{v,m,DV}} - \mathbf{v}_{\mathrm{c}}\right) + \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu \\ &= \tilde{s}(U_{\mathrm{d,m}}\cos\chi_{\mathrm{r}} - U_{\mathrm{c}}) + \tilde{e}U_{\mathrm{d,m}}\sin\chi_{\mathrm{r}} + \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu,\end{aligned}$$

where $\boldsymbol{\alpha}_{\mathrm{v,m,DV}} = [U_{\mathrm{d,m}}, 0]^\top$. Hence, choose U_{c} as

$$U_{\mathrm{c}} = U_{\mathrm{d,m}}\cos\chi_{\mathrm{r}} + \gamma\tilde{s} \tag{31}$$

with $\gamma > 0$ constant, and χ_{r} as the helmsman-like

$$\chi_{\mathrm{r}} = \arctan\left(-\frac{\tilde{e}}{\triangle_{\tilde{\mathrm{e}}}}\right) \tag{32}$$

with $\triangle_{\tilde{\mathrm{e}}} > 0$ (not necessarily constant), giving

$$\dot{V}_{\tilde{\varepsilon}} = -\gamma\tilde{s}^2 - U_{\mathrm{d,m}}\frac{\tilde{e}^2}{\sqrt{\tilde{e}^2 + \triangle_{\tilde{\mathrm{e}}}^2}} + \tilde{\varepsilon}^\top \mathbf{R}_{\mathrm{C}}^\top \mathbf{R}_{\mathrm{B}}\mathbf{H}\mathbf{z}_\nu. \tag{33}$$

Consequently

$$\dot{\varpi}_{\mathrm{c}} = \frac{U_{\mathrm{c}}}{|\mathbf{p}_{\mathrm{c}}'|} \tag{34}$$

and

$$\chi_{\mathrm{d}} = \chi_{\mathrm{c}} + \chi_{\mathrm{r}}, \tag{35}$$

with U_{c} as in (31), χ_{c} as in (19), and χ_{r} as in (32). Hence, the collaborator tracks the mediator instead of the surface craft itself. Also, the desired linear velocity $\boldsymbol{\alpha}_{\mathrm{v,m}}$ must be computed by

$$\begin{aligned}\boldsymbol{\alpha}_{\mathrm{v,m}} &= \mathbf{R}_{\mathrm{B}}^\top \mathbf{R}_{\mathrm{DV}}\boldsymbol{\alpha}_{\mathrm{v,m,DV}} \\ &= \begin{bmatrix} U_{\mathrm{d,m}}\cos(\chi_{\mathrm{d}} - \psi) \\ U_{\mathrm{d,m}}\sin(\chi_{\mathrm{d}} - \psi) \end{bmatrix},\end{aligned} \tag{36}$$

which gives the desired linear velocity $\boldsymbol{\alpha}_{\mathrm{v}}$ through the relationship

$$\boldsymbol{\alpha}_{\mathrm{v}} = \boldsymbol{\alpha}_{\mathrm{v,m}} + \mathbf{R}_{\mathrm{B}}^\top \mathbf{R}_{\mathrm{C}}(\mathbf{S}_{\mathrm{C}}\varepsilon_{\mathrm{f}} + \dot{\varepsilon}_{\mathrm{f}}). \tag{37}$$

Subsequently, write the system dynamics of $\tilde{\varepsilon}$ and $\mathbf{z}_{\mathrm{g}}$

$$\Sigma_1 : \dot{\tilde{\varepsilon}} = \mathbf{f}_1(t, \tilde{\varepsilon}) + \mathbf{g}_1(t, \tilde{\varepsilon}, \mathbf{z}_\mathrm{g})\mathbf{z}_\mathrm{g} \tag{38}$$

$$\Sigma_2 : \dot{\mathbf{z}}_\mathrm{g} = \mathbf{f}_2(t, \mathbf{z}_\mathrm{g}), \tag{39}$$

which is a pure cascade where the control subsystem perturbs the guidance subsystem through the interconnection matrix

$$\mathbf{g}_1(t, \tilde{\varepsilon}, \mathbf{z}_\mathrm{g}) = \left[\mathbf{0}_{2\times 1}, \mathbf{R}_\mathrm{C}^\top \mathbf{R}_\mathrm{B}\mathbf{H}, \mathbf{0}_{2\times 3}\right]. \tag{40}$$

Also, consider the following assumptions

Assumption 2. $\left|\mathbf{p}'_\mathrm{p}\right| \in \left[\left|\mathbf{p}'_\mathrm{p}\right|_{\min}, \left|\mathbf{p}'_\mathrm{p}\right|_{\max}\right] \forall \varpi \in \mathbb{R}$

Assumption 3. $\triangle_{\tilde{\mathrm{e}}} \in [\triangle_{\tilde{\mathrm{e}},\min}, \infty\rangle$, $\triangle_{\tilde{\mathrm{e}},\min} > 0$

Assumption 4. $U_{\mathrm{d,m}} \in [U_{\mathrm{d,m,min}}, \infty\rangle$, $U_{\mathrm{d,m,min}} > 0$

By contemplating $\boldsymbol{\xi} = \left[\tilde{\varepsilon}^\top, \mathbf{z}_\mathrm{g}^\top\right]^\top$, we arrive at

Proposition 5. The equilibrium point $\boldsymbol{\xi} = \mathbf{0}$ is rendered uniformly globally asymptotically and locally exponentially stable (UGAS/ULES) under assumptions (2-4) when applying (13-14) with the reference signals (12) and (37).

PROOF. Since the origin of system Σ_2 is shown to be UGAS/ULES in Proposition 1, the origin of the unperturbed system Σ_1 (i.e., when $\mathbf{z}_\mathrm{g} = \mathbf{0}$) is trivially shown to be UGAS/ULES by applying standard Lyapunov theory to (16) and (33), and the interconnection term satisfies $\left|\mathbf{g}_1(t, \boldsymbol{\varepsilon}, \mathbf{z}_\mathrm{g})\right| = 1$, the proposed result follows directly from Theorem 7 and Lemma 8 of (Panteley *et al.* 1998).

Note that the stability property of Proposition 5 is also called global κ-exponential stability (Sørdalen and Egeland 1995).

2.3.3. Step 3: Synchronization Loop Design In this final design step, we determine the required *size* of $\boldsymbol{\alpha}_\mathrm{v}$, derived indirectly through $U_{\mathrm{d,m}}$, such that a surface craft controlled by (13) and (14) with reference signals given by (12) and (37) synchronize with the formation leader. Consequently, consider the positive definite and radially unbounded CLF

$$V_{\tilde{\varpi}} = \frac{1}{2}\tilde{\varpi}^2, \tag{41}$$

where

$$\tilde{\varpi} = \varpi_\mathrm{c} - \varpi_\mathrm{l}, \tag{42}$$

and differentiate the CLF with respect to time to get

$$\begin{aligned} \dot{V}_{\tilde{\varpi}} &= \tilde{\varpi}\dot{\tilde{\varpi}} \\ &= \tilde{\varpi}(\dot{\varpi}_\mathrm{c} - \dot{\varpi}_\mathrm{l}) \\ &= \tilde{\varpi}(z_\mathrm{c} + \alpha_\mathrm{c} - \dot{\varpi}_\mathrm{l}) \end{aligned}$$

where $z_\mathrm{c} = \dot{\varpi}_\mathrm{c} - \alpha_\mathrm{c}$, and α_c represents the desired speed of the collaborator when the mediator has converged to the path, i.e., when $\boldsymbol{\xi} = \mathbf{0}$. Hence, we have that

$$\dot{V}_{\tilde{\varpi}} = \tilde{\varpi}\left(\frac{U_{\mathrm{d,m}}}{|\mathbf{p}'_\mathrm{c}|} - \frac{U_\mathrm{l}}{|\mathbf{p}'_\mathrm{l}|}\right) + \tilde{\varpi}z_\mathrm{c},$$

which is equal to

$$\dot{V}_{\tilde{\varpi}} = -k_{\tilde{\varpi}}\frac{\tilde{\varpi}^2}{\sqrt{\tilde{\varpi}^2 + \triangle_{\tilde{\varpi}}^2}} + \tilde{\varpi}z_\mathrm{c} \tag{43}$$

when choosing

$$U_{\mathrm{d,m}} = |\mathbf{p}'_\mathrm{c}|\left(\frac{U_\mathrm{l}}{|\mathbf{p}'_\mathrm{l}|} - k_{\tilde{\varpi}}\frac{\tilde{\varpi}}{\sqrt{\tilde{\varpi}^2 + \triangle_{\tilde{\varpi}}^2}}\right), \tag{44}$$

where $\triangle_{\tilde{\varpi}} \in [\triangle_{\tilde{\varpi},\min}, \infty\rangle$, $\triangle_{\tilde{\varpi},\min} > 0$, and where

$$k_{\tilde{\varpi}} = \sigma\frac{U_\mathrm{l}}{|\mathbf{p}'_\mathrm{l}|}, \sigma \in \langle 0, 1] \tag{45}$$

ensures that $U_{\mathrm{d,m}}$ satisfies Assumption 4 by leading to

$$U_{\mathrm{d,m}} = U_\mathrm{l}\left(1 - \sigma\frac{\tilde{\varpi}}{\sqrt{\tilde{\varpi}^2 + \triangle_{\tilde{\varpi}}^2}}\right)\frac{|\mathbf{p}'_\mathrm{c}|}{|\mathbf{p}'_\mathrm{l}|}. \tag{46}$$

Then, expand z_c to get

$$\begin{aligned} z_\mathrm{c} &= \dot{\varpi}_\mathrm{c} - \alpha_\mathrm{c} \\ &= \frac{U_{\mathrm{d,m}}(\cos\chi_\mathrm{r} - 1) + \gamma s}{|\mathbf{p}'_\mathrm{c}|} \end{aligned} \tag{47}$$

where

$$(\cos\chi_\mathrm{r} - 1) = \frac{\triangle_{\tilde{\mathrm{e}}} - \sqrt{\tilde{e}^2 + \triangle_{\tilde{\mathrm{e}}}^2}}{\sqrt{\tilde{e}^2 + \triangle_{\tilde{\mathrm{e}}}^2}} \tag{48}$$

such that the system dynamics of $\tilde{\varpi}$ and $\boldsymbol{\xi}$ becomes

$$\Sigma_3 : \dot{\tilde{\varpi}} = f_3(t, \tilde{\varpi}) + \mathbf{g}_3(t, \tilde{\varpi}, \boldsymbol{\xi})^\top\boldsymbol{\xi} \tag{49}$$

$$\Sigma_4 : \dot{\boldsymbol{\xi}} = \mathbf{f}_4(t, \boldsymbol{\xi}), \tag{50}$$

which shows that the control and guidance subsystems perturb the synchronization subsystem through

$$\mathbf{g}_3(t, \tilde{\varpi}, \boldsymbol{\xi}) = \frac{1}{|\mathbf{p}'_\mathrm{c}|}\left[\gamma, U_{\mathrm{d,m}}\frac{\triangle_{\tilde{\mathrm{e}}} - \sqrt{\tilde{e}^2 + \triangle_{\tilde{\mathrm{e}}}^2}}{\tilde{e}\sqrt{\tilde{e}^2 + \triangle_{\tilde{\mathrm{e}}}^2}}, \mathbf{0}_{1\times 7}\right]^\top, \tag{51}$$

which is well-defined since

$$\lim_{\tilde{\mathrm{e}}\to 0}\frac{\triangle_{\tilde{\mathrm{e}}} - \sqrt{\tilde{e}^2 + \triangle_{\tilde{\mathrm{e}}}^2}}{\tilde{e}\sqrt{\tilde{e}^2 + \triangle_{\tilde{\mathrm{e}}}^2}} = 0. \tag{52}$$

Considering $\boldsymbol{\zeta} = \left[\tilde{\varpi}, \boldsymbol{\xi}^\top\right]^\top$, we can now state

Theorem 6. The equilibrium point $\boldsymbol{\zeta} = \mathbf{0}$ is rendered UGAS/ULES under assumptions (2-3) when applying (13-14) with reference signals (12) and (37) with (46).

PROOF. Since the origin of system Σ_4 is shown to be UGAS/ULES in Proposition 5, the origin of the unperturbed system Σ_3 (i.e., when $\boldsymbol{\xi} = \mathbf{0}$) is trivially shown to be UGAS/ULES by applying standard Lyapunov theory to (41) and (43), and the interconnection term satisfies $|\mathbf{g}_3(t, \tilde{\varpi}, \boldsymbol{\xi})| < \left|\mathbf{p}'_\mathrm{p}\right|_{\min}^{-1}\left(\gamma^2 + \left(\frac{U_{\mathrm{l,max}}}{\triangle_{\tilde{\mathrm{e}},\min}}\right)^2\right)^{1/2}$, the proposed result follows directly from Theorem 7 and Lemma 8 of (Panteley *et al.* 1998).

If every formation member is able to satisfy the conditions of Theorem 6, the formation assembles to solve the formation control problem as stated in (6).

3. DISCUSSION

The guided scheme can be extended to handle underactuated surface craft due to its output space of linear velocity and heading, where the heading can either be independently controlled (fully actuated case) or dedicated to control the orientation of the linear velocity (underactuated case). Hence, keeping the control subsystem unchanged, a redesign of the guidance and synchronization subsystems can enable underactuated operations. In fact, when the scenario entails straight-line paths and no environmental disturbances, a redesign is not necessary if the desired heading is assigned as the desired orientation of the linear velocity. Furthermore, the specific version of the guided scheme treated in this paper is completely decentralized; no coordination variables need be communicated. Hence, the loop is open at the leader-follower level, i.e., the leader propagates without feedback from the followers. Consequently, while impervious to single-point failure, the formation suffers from graceful degradation, i.e., members who cannot keep up with the leader fall out of formation. However, they will still be able to follow their assigned formation positions relative to the path. Thus, this specific scheme could be classified as involving tactical (i.e., local/individual) path following, but strategic (i.e., global/formation-wide) trajectory tracking. An alternative solution involves path following at both the tactical and strategic levels, where the leader receives formation-wide feedback from the followers.

4. CONCLUSIONS

This paper has considered the topic of formation control for fully actuated marine surface craft. A so-called guided formation control concept has been developed within a leader-follower framework by means of integrator backstepping design and theory on nonlinear time-varying cascades. A basic assumption of the approach is that the formation control designer can choose the path to be traversed, while key qualities encompass helmsman-like transient motion behavior and extendability toward underactuated vehicles.

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