MOTION CONTROL CONCEPTS FOR TRAJECTORY TRACKING OF FULLY ACTUATED SHIPS

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Abstract: This paper develops and compares two different motion control concepts for fully actuated ships in a trajectory tracking scenario. The first concept, named servoed motion control, is identical to a standard solution found in the established ship control literature on dynamic positioning. The scheme is easy to derive and analyze by traditional nonlinear control theory, but not readily extendable to underactuated ships, while also exhibiting erratic transient convergence behavior. The second concept, named guided motion control, represents a novel approach inspired by literature on missile guidance, path following, and formation control. The guided concept is more complex to derive and analyze, but is readily extendable to underactuated ships, and displays gentle transient convergence behavior.

Keywords: Ship motion control, Fully actuated ships, Trajectory tracking scenario, Servoed motion control, Guided motion control

1. INTRODUCTION

Motion control is a fundamental enabling technology for any ship application. Whether humans or computers act as motion controllers, a ship requires such a tactical level to translate strategic motion planning commands into physically realistic movements of the ship hull. Today, co-control is probably the most common configuration for industrial applications, i.e., that humans maneuver ships through sail-by-wire technology, where computers are responsible for (tactical-level) motion control while humans take care of the (strategic-level) motion planning. However, sensitivity toward human casualties when operating in so-called dirty, dull, and dangerous environments will gradually enforce full autonomy requirements. Likewise, full autonomy is attractive through prospects of increased operational cost saving, endurance, precision, reliability, and safety within marine business areas such as shipping, offshore, and fisheries and aquaculture. But full autonomy requires extraordinary features, not least at the tactical level of motion control. Hence, this work considers two qualitatively different motion control concepts, and debates their distinctive qualities.

The main contribution of this paper is the development of a motion control concept denoted *guided motion control*. In a three-step, backstepping-inspired, cascaded-based design, a novel motion controller is developed that draws on control, guidance, and synchronization concepts. The approach is inspired by schemes from research areas such as missile guidance, path following, and formation control. For a trajectory tracking scenario concerning fully actuated ships, the guided concept is contrasted toward what we have called the servoed motion control concept, which represents a standard solution in the established literature on nonlinear motion control for fully actuated ships.

Since the goal of this work is to emphasize and illustrate fundamental aspects of motion control, a simplified ship model that does not consider environmental disturbances is employed. Hence, the paper should be regarded purely as a motion control concept study.

Notation: The time derivative of (a vector) $\mathbf{x}(t)$ is denoted $\dot{\mathbf{x}}$, the partial derivative of $\mathbf{x}(\varpi(t))$ is denoted $\mathbf{x}'\ \left(= \frac{\partial \mathbf{x}}{\partial \varpi}(\varpi(t))\right)$, while $|\cdot|$ represents the Euclidean vector norm as well as the induced matrix norm.

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2. MOTION CONTROL CONCEPTS

This section develops two different motion control concepts that can be applied for fully actuated ships in a trajectory tracking scenario. The applied ship model is simplified such that we can concentrate on the fundamental aspects of motion control. However, any standard solutions to bias counteraction, wave filtering, and control allocation are readily applicable for fully actuated ships that employ one of the motion control concepts under consideration.

2.1 Dynamic Ship Model

A 3 degree-of-freedom (DOF) dynamic model of a ship (surface vessel; surge, sway, and yaw modes) can be found in (Fossen 2002), and is composed of the kinematics

$$\dot{\boldsymbol{\eta}} = \mathbf{R}(\psi)\boldsymbol{\nu}, \tag{1}$$

and the (purposefully simplified) kinetics

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} = \boldsymbol{\tau}, \tag{2}$$

where $\boldsymbol{\eta} = [x, y, \psi]^\top \in \mathbb{R}^2 \times \mathcal{S}$ represents the earth-fixed position and heading (with $\mathcal{S} = [-\pi, \pi]$), $\boldsymbol{\nu} = [u, v, r]^\top \in \mathbb{R}^3$ represents the vessel-fixed velocity, $\mathbf{R}(\psi) \in SO(3)$ is the transformation matrix

$$\mathbf{R}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3}$$

that transforms from the vessel-fixed BODY frame (**B**) to the earth-fixed NED frame (**N**), $\mathbf{M}$ is the inertia matrix, $\mathbf{C}(\boldsymbol{\nu})$ is the centrifugal and coriolis matrix, while $\mathbf{D}(\boldsymbol{\nu})$ is the hydrodynamic damping matrix. The system matrices satisfy the properties $\mathbf{M} = \mathbf{M}^\top > 0$, $\mathbf{C} = -\mathbf{C}^\top$ and $\mathbf{D} > 0$. The vessel-fixed propulsion forces and moment is represented by $\boldsymbol{\tau} = [\tau_\mathrm{X}, \tau_\mathrm{Y}, \tau_\mathrm{N}]^\top \in \mathbb{R}^3$. A fully actuated ship can independently control all 3 DOFs simultaneously, which means that the direction of the linear velocity is independent of the heading. This is not the case for an underactuated ship, where the orientation of the linear velocity is coupled, in a sense locked, to the heading of the ship.

2.2 Trajectory Tracking Scenario

A fundamental assumption of this paper is that the motion control designer is granted the freedom to design the path (trajectory) that is to be traversed, as well as the motion of a target point that is to be tracked along the path. Consequently, consider a (planar) path continuously parameterized by a scalar variable $\varpi \in \mathbb{R}$, such that the position of a point belonging to the path is represented by $\mathbf{p}_\mathrm{p}(\varpi) \in \mathbb{R}^2$. Thus, the path is a one-dimensional manifold that can be expressed by the set

$$\mathcal{P} = \left\{\mathbf{p} \in \mathbb{R}^2 \mid \mathbf{p} = \mathbf{p}_\mathrm{p}(\varpi)\ \forall \varpi \in \mathbb{R}\right\}. \tag{4}$$

Then, consider a time-varying target point $\mathbf{p}_\mathrm{t}(t) = \mathbf{p}_\mathrm{p}(\varpi_\mathrm{t}(t))$ traversing the path by adhering to a chosen speed profile $U_\mathrm{t}(\varpi_\mathrm{t})$, implemented through

$$\dot{\varpi}_\mathrm{t} = \frac{U_\mathrm{t}(\varpi_\mathrm{t})}{\left|\mathbf{p}'_\mathrm{p}(\varpi_\mathrm{t})\right|}, \tag{5}$$

since $|\dot{\mathbf{p}}_\mathrm{t}| = \left|\mathbf{p}'_\mathrm{p}(\varpi_\mathrm{t})\right| \dot{\varpi}_\mathrm{t} = U_\mathrm{t}(\varpi_\mathrm{t})$, where $U_\mathrm{t}(\varpi_\mathrm{t}) \in [U_{\mathrm{t,min}}, U_{\mathrm{t,max}}]$, $U_{\mathrm{t,min}} > 0$. The path, as well as the speed profile of the target point, is typically designed (by offline optimization) based on information like the nominal actuator configuration of the ship in question, weather forecasts, static obstacles, etc. A trajectory tracking (path tracking) scenario entails that the spatial and temporal assignments are linked as one single assignment, requiring that the ship is located at a certain point along the path at a certain time. The target point drives the ship-path system forward (as an independent, time-varying attractor moving along the path) by propagating without concern for the tracking ship. Thus, the ship-path system is inherently open loop, and the target may leave the ship behind if something should alter the propulsive capability of the ship.

2.2.1. Problem Statement In our scenario, the trajectory tracking problem for a fully actuated ship can be stated by

$$\lim_{t\to\infty} (\boldsymbol{\eta}(t) - \boldsymbol{\eta}_\mathrm{d}(t)) = \mathbf{0}, \tag{6}$$

where $\boldsymbol{\eta}_\mathrm{d}(t) = \left[\mathbf{p}_\mathrm{t}^\top(t), \psi_\mathrm{d}(t)\right]^\top$, and where $\psi_\mathrm{d}(t)$ can be any arbitrary heading, satisfying for instance some auxiliary task objective.

2.3 Servoed Motion Control

Fully actuated ships nominally imply low-speed dynamic positioning (DP) applications. This relates to the fact that the ability to produce a transversal (sway) force is lost when moving too fast. The DP literature has become rich and varied, and during the last decade nonlinear control theory has been applied to create motion controllers ensuring global stability properties. These controllers are inspired by work performed in the robotics community, which is readily extendable to ships when formulating their dynamic model within the Euler-Lagrange framework (Fossen 2002). The concept of this section has been termed servoed motion control due to the underlying control design principle, which resembles that of basic servomechanisms. Hence, the control objective is identical to the problem statement, and straightforwardly achieved in one single design step. Unfortunately, a consequence of employing such a servoed design principle is that the resulting motion controllers cannot be readily modified to handle underactuated vehicles.

The derived motion controller is backstepping-based (Krstić *et al.* 1995), and standard in the DP control literature (Fossen 2002). Consequently, consider the positive definite and radially unbounded Control Lyapunov Function (CLF)

$$V_{\mathrm{s}} = \frac{1}{2}(\mathbf{z}_\eta^\top \mathbf{z}_\eta + \mathbf{z}_\nu^\top \mathbf{M}\mathbf{z}_\nu), \tag{7}$$

where we have defined

$$\mathbf{z}_\eta = \mathbf{R}^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_{\mathrm{d}}) \tag{8}$$

with $\mathbf{R} = \mathbf{R}(\psi)$, and

$$\mathbf{z}_\nu = \boldsymbol{\nu} - \boldsymbol{\alpha}, \tag{9}$$

where $\boldsymbol{\alpha} = [\alpha_u, \alpha_v, \alpha_r]^\top \in \mathbb{R}^3$ is a so-called vector of stabilizing functions (virtual inputs that become reference signals) yet to be designed. Then, differentiate the CLF along the trajectories of $\mathbf{z}_\eta$ and $\mathbf{z}_\nu$ to obtain

$$\begin{aligned}
\dot{V}_{\mathrm{s}} &= \mathbf{z}_\eta^\top \dot{\mathbf{z}}_\eta + \mathbf{z}_\nu^\top \mathbf{M}\dot{\mathbf{z}}_\nu \\
&= \mathbf{z}_\eta^\top(\dot{\mathbf{R}}^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_{\mathrm{d}}) + \mathbf{R}^\top(\dot{\boldsymbol{\eta}} - \dot{\boldsymbol{\eta}}_{\mathrm{d}})) + \\
&\quad \mathbf{z}_\nu^\top \mathbf{M}(\dot{\boldsymbol{\nu}} - \dot{\boldsymbol{\alpha}}) \\
&= \mathbf{z}_\eta^\top(\mathbf{S}^\top\mathbf{R}^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_{\mathrm{d}}) + \mathbf{R}^\top\dot{\boldsymbol{\eta}} - \mathbf{R}^\top\dot{\boldsymbol{\eta}}_{\mathrm{d}}) + \\
&\quad \mathbf{z}_\nu^\top(\mathbf{M}\dot{\boldsymbol{\nu}} - \mathbf{M}\dot{\boldsymbol{\alpha}}) \\
&= \mathbf{z}_\eta^\top(\mathbf{S}^\top\mathbf{z}_\eta + \boldsymbol{\nu} - \mathbf{R}^\top\dot{\boldsymbol{\eta}}_{\mathrm{d}}) + \\
&\quad \mathbf{z}_\nu^\top(\boldsymbol{\tau} - \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} - \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} - \mathbf{M}\dot{\boldsymbol{\alpha}})
\end{aligned}$$

since $\dot{\mathbf{R}} = \mathbf{R}\mathbf{S}$ with $\mathbf{S} = -\mathbf{S}^\top$. Furthermore, recognizing that $\mathbf{z}_\eta^\top\mathbf{S}^\top\mathbf{z}_\eta = 0$ and $\boldsymbol{\nu} = \mathbf{z}_\nu + \boldsymbol{\alpha}$, we get

$$\begin{aligned}
\dot{V}_{\mathrm{s}} &= \mathbf{z}_\eta^\top(\boldsymbol{\alpha} - \mathbf{R}^\top\dot{\boldsymbol{\eta}}_{\mathrm{d}}) + \\
&\quad \mathbf{z}_\nu^\top(\boldsymbol{\tau} - \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} - \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} - \mathbf{M}\dot{\boldsymbol{\alpha}} + \mathbf{z}_\eta),
\end{aligned}$$

which results in

$$\begin{aligned}
\dot{V}_{\mathrm{s}} &= -\mathbf{z}_\eta^\top\mathbf{K}_\eta\mathbf{z}_\eta - \mathbf{z}_\nu^\top\mathbf{C}(\boldsymbol{\nu})\mathbf{z}_\nu - \mathbf{z}_\nu^\top\mathbf{D}(\boldsymbol{\nu})\mathbf{z}_\nu + \\
&\quad \mathbf{z}_\nu^\top(\boldsymbol{\tau} - \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\alpha} - \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\alpha} - \mathbf{M}\dot{\boldsymbol{\alpha}} + \mathbf{z}_\eta)
\end{aligned}$$

when choosing the virtual input as

$$\boldsymbol{\alpha} = \mathbf{R}^\top\dot{\boldsymbol{\eta}}_{\mathrm{d}} - \mathbf{K}_\eta\mathbf{z}_\eta, \tag{10}$$

where $\mathbf{K}_\eta = \mathbf{K}_\eta^\top > 0$ is a constant matrix. Finally, since $\mathbf{z}_\nu^\top\mathbf{C}(\boldsymbol{\nu})\mathbf{z}_\nu = 0$, and by selecting the control input as

$$\boldsymbol{\tau} = \mathbf{M}\dot{\boldsymbol{\alpha}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\alpha} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\alpha} - \mathbf{z}_\eta - \mathbf{K}_\nu\mathbf{z}_\nu, \tag{11}$$

where $\mathbf{K}_\nu = \mathbf{K}_\nu^\top > 0$ is a constant matrix, we obtain the quadratically negative definite

$$\dot{V}_{\mathrm{s}} = -\mathbf{z}_\eta^\top\mathbf{K}_\eta\mathbf{z}_\eta - \mathbf{z}_\nu^\top(\mathbf{D}(\boldsymbol{\nu}) + \mathbf{K}_\nu)\mathbf{z}_\nu. \tag{12}$$

By considering the state vector $\mathbf{z}_{\mathrm{s}} = \left[\mathbf{z}_\eta^\top, \mathbf{z}_\nu^\top\right]^\top$, we can now state the following theorem

Theorem 1. (Servoed Motion Control). The equilibrium point $\mathbf{z}_{\mathrm{s}} = \mathbf{0}$ is rendered uniformly globally exponentially stable (UGES) by adhering to (10) and (11) when $\boldsymbol{\eta}_{\mathrm{d}}$, $\dot{\boldsymbol{\eta}}_{\mathrm{d}}$ and $\ddot{\boldsymbol{\eta}}_{\mathrm{d}}$ are uniformly bounded.

PROOF. By standard Lyapunov theory, (7) and (12) show that the origin of $\mathbf{z}_{\mathrm{s}}$ is UGES.

Consequently, the trajectory tracking problem as stated in (6) has been solved in one design step.

2.4 Guided Motion Control

This section solves the trajectory tracking problem in three distinct design steps by a backstepping-inspired, cascaded-based procedure that is influenced by ideas from missile guidance (Shneydor 1998), path following (Breivik and Fossen 2005), and formation control (Breivik *et al.* 2006). The underlying concept is named guided motion control since it allows the transient convergence behavior to be manipulated through guidance laws. An advantage of employing a guided design principle is that the resulting motion control structure can be readily modified to also handle underactuated vehicles.

2.4.1. Step 1: Control Loop Design Since the position of a ship can be controlled through its linear velocity, we redefine the output space from the 3 DOF position and heading to the 3 DOF linear velocity and heading. Again, we design the controller by using the backstepping approach. Consequently, consider the positive definite and radially unbounded CLF

$$V_{\mathrm{g}} = \frac{1}{2}(z_\psi^2 + \mathbf{z}_\nu^\top\mathbf{M}\mathbf{z}_\nu) \tag{13}$$

where we have

$$z_\psi = \psi - \psi_{\mathrm{d}} \tag{14}$$

and

$$\mathbf{z}_\nu = \boldsymbol{\nu} - \boldsymbol{\alpha}, \tag{15}$$

where $\boldsymbol{\alpha}$ is yet to be designed. Subsequently, differentiate the CLF with respect to time to obtain

$$\begin{aligned}
\dot{V}_{\mathrm{g}} &= z_\psi\dot{z}_\psi + \mathbf{z}_\nu^\top\mathbf{M}\dot{\mathbf{z}}_\nu \\
&= z_\psi(\dot{\psi} - \dot{\psi}_{\mathrm{d}}) + \mathbf{z}_\nu^\top\mathbf{M}(\dot{\boldsymbol{\nu}} - \dot{\boldsymbol{\alpha}}) \\
&= z_\psi(\mathbf{h}^\top\dot{\boldsymbol{\eta}} - \dot{\psi}_{\mathrm{d}}) + \mathbf{z}_\nu^\top(\mathbf{M}\dot{\boldsymbol{\nu}} - \mathbf{M}\dot{\boldsymbol{\alpha}})
\end{aligned}$$

where

$$\mathbf{h} = [0, 0, 1]^\top. \tag{16}$$

Then, recognizing that $\mathbf{h}^\top\dot{\boldsymbol{\eta}} = \mathbf{h}^\top\mathbf{R}\boldsymbol{\nu} = \mathbf{h}^\top\boldsymbol{\nu}$ and $\boldsymbol{\nu} = \mathbf{z}_\nu + \boldsymbol{\alpha}$, we obtain

$$\begin{aligned}
\dot{V}_{\mathrm{g}} &= z_\psi(\mathbf{h}^\top\boldsymbol{\alpha} - \dot{\psi}_{\mathrm{d}}) + \\
&\quad \mathbf{z}_\nu^\top(\boldsymbol{\tau} - \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} - \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} - \mathbf{M}\dot{\boldsymbol{\alpha}} + \mathbf{h}z_\psi),
\end{aligned}$$

which results in

$$\begin{aligned}
\dot{V}_{\mathrm{g}} &= -k_\psi z_\psi^2 - \mathbf{z}_\nu^\top\mathbf{C}(\boldsymbol{\nu})\mathbf{z}_\nu - \mathbf{z}_\nu^\top\mathbf{D}(\boldsymbol{\nu})\mathbf{z}_\nu + \\
&\quad \mathbf{z}_\nu^\top(\boldsymbol{\tau} - \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\alpha} - \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\alpha} - \mathbf{M}\dot{\boldsymbol{\alpha}} + \mathbf{h}z_\psi)
\end{aligned}$$

when choosing the virtual input $\mathbf{h}^\top\boldsymbol{\alpha} = \alpha_{\mathrm{r}}$ as

$$\alpha_{\mathrm{r}} = \dot{\psi}_{\mathrm{d}} - k_\psi z_\psi, \tag{17}$$

where $k_\psi > 0$ is a constant. Since $\mathbf{z}_\nu^\top \mathbf{C}(\boldsymbol{\nu})\mathbf{z}_\nu = 0$, and by selecting the control input as

$$\boldsymbol{\tau} = \mathbf{M}\dot{\boldsymbol{\alpha}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\alpha} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\alpha} - \mathbf{h}z_\psi - \mathbf{K}_\nu \mathbf{z}_\nu, \quad (18)$$

where $\mathbf{K}_\nu = \mathbf{K}_\nu^\top > 0$ is a constant matrix, we finally obtain the quadratically negative definite

$$\dot{V}_\mathrm{g} = -k_\psi z_\psi^2 - \mathbf{z}_\nu^\top(\mathbf{D}(\boldsymbol{\nu}) + \mathbf{K}_\nu)\mathbf{z}_\nu. \quad (19)$$

Considering the state vector $\mathbf{z}_\mathrm{g} = \left[z_\psi, \mathbf{z}_\nu^\top\right]^\top$, the following proposition can now be stated

Proposition 2. The equilibrium point $\mathbf{z}_\mathrm{g} = \mathbf{0}$ is rendered uniformly globally exponentially stable (UGES) by adhering to (17) and (18) under the assumption that $\boldsymbol{\alpha}$ and $\dot{\boldsymbol{\alpha}}$ are uniformly bounded.

PROOF. By standard Lyapunov theory, (13) and (19) show that the origin of $\mathbf{z}_\mathrm{g}$ is UGES.

Unlike the result in Theorem 1, Proposition 2 does not mean that the trajectory tracking problem as stated in (6) has been solved. In fact, the controller that has been developed here cannot achieve anything meaningful unless it is fed sensible reference signals, i.e., unless $\boldsymbol{\alpha}_\mathrm{v} = [\alpha_\mathrm{u}, \alpha_\mathrm{v}]^\top \in \mathbb{R}^2$ is purposefully defined for the problem at hand. This, then, represents the challenge for the final two design steps.

2.4.2. Step 2: Guidance Loop Design Here, we design the required *orientation* of $\boldsymbol{\alpha}_\mathrm{v}$ ($|\boldsymbol{\alpha}_\mathrm{v}| > 0$) such that a ship controlled by (18) achieves path following. The design is inspired by the concept of guidance-based path following as found in (Breivik and Fossen 2005). Consequently, consider the positive definite and radially unbounded CLF

$$V_\varepsilon = \frac{1}{2}\boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon}, \quad (20)$$

with

$$\boldsymbol{\varepsilon} = \mathbf{R}_\mathrm{C}^\top(\mathbf{p} - \mathbf{p}_\mathrm{c}) \quad (21)$$

where $\mathbf{p}_\mathrm{c} = \mathbf{p}_\mathrm{p}(\varpi_\mathrm{c})$ represents a *collaborator* point that acts cooperatively with the ship as an intermediate path attractor, and whose sole purpose is to ensure that the ship can converge to the path even if it has not converged to the target point. For a given ϖ_c, define a path-tangential reference frame at $\mathbf{p}_\mathrm{c}$ termed the COLLABORATOR frame (**C**). To arrive at **C**, the INERTIAL frame (**I**) must be positively rotated an angle

$$\chi_\mathrm{c} = \arctan\left(\frac{y_\mathrm{p}'(\varpi_\mathrm{c})}{x_\mathrm{p}'(\varpi_\mathrm{c})}\right), \quad (22)$$

which can be represented by the rotation matrix

$$\mathbf{R}_\mathrm{C} = \begin{bmatrix} \cos\chi_\mathrm{c} & -\sin\chi_\mathrm{c} \\ \sin\chi_\mathrm{c} & \cos\chi_\mathrm{c} \end{bmatrix}, \quad (23)$$

$\mathbf{R}_\mathrm{C} \in SO(2)$. Hence, equation (21) represents the error vector between the ship and its collaborator decomposed in **C**. The local coordinates $\boldsymbol{\varepsilon} = [s, e]^\top$ consist of the along-track error s and the cross-track error e. It is clear that path following can be achieved by driving $\boldsymbol{\varepsilon}$ to zero. Thus, differentiate the CLF in (20) along the trajectories of $\boldsymbol{\varepsilon}$ to obtain

$$\begin{aligned}\dot{V}_\varepsilon &= \boldsymbol{\varepsilon}^\top\dot{\boldsymbol{\varepsilon}} \\ &= \boldsymbol{\varepsilon}^\top(\mathbf{S}_\mathrm{C}^\top\mathbf{R}_\mathrm{C}^\top(\mathbf{p}-\mathbf{p}_\mathrm{c}) + \mathbf{R}_\mathrm{C}^\top(\dot{\mathbf{p}} - \dot{\mathbf{p}}_\mathrm{c})) \\ &= \boldsymbol{\varepsilon}^\top\left(\mathbf{S}_\mathrm{C}^\top\boldsymbol{\varepsilon} + \mathbf{R}_\mathrm{C}^\top\mathbf{H}\dot{\boldsymbol{\eta}} - \mathbf{v}_\mathrm{c}\right) \\ &= \boldsymbol{\varepsilon}^\top\left(\mathbf{R}_\mathrm{C}^\top\mathbf{H}\mathbf{R}\boldsymbol{\nu} - \mathbf{v}_\mathrm{c}\right)\end{aligned}$$

where $\mathbf{S}_\mathrm{C} = -\mathbf{S}_\mathrm{C}^\top \Rightarrow \boldsymbol{\varepsilon}^\top\mathbf{S}_\mathrm{C}^\top\boldsymbol{\varepsilon} = 0$, and where

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}. \quad (24)$$

Also, $\mathbf{v}_\mathrm{c} = [U_\mathrm{c}, 0]^\top$, where $U_\mathrm{c} = |\dot{\mathbf{p}}_\mathrm{c}|$, represents the linear velocity of the collaborator decomposed in **C**. Furthermore, we have that

$$\begin{aligned}\dot{V}_\varepsilon &= \boldsymbol{\varepsilon}^\top\left(\mathbf{R}_\mathrm{C}^\top\mathbf{H}\mathbf{R}\boldsymbol{\alpha} - \mathbf{v}_\mathrm{c}\right) + \boldsymbol{\varepsilon}^\top\mathbf{R}_\mathrm{C}^\top\mathbf{H}\mathbf{R}\mathbf{z}_\nu \\ &= \boldsymbol{\varepsilon}^\top\left(\mathbf{R}_\mathrm{C}^\top\mathbf{H}\mathbf{R}\mathbf{H}^\top\mathbf{H}\boldsymbol{\alpha} - \mathbf{v}_\mathrm{c}\right) + \boldsymbol{\varepsilon}^\top\mathbf{R}_\mathrm{C}^\top\mathbf{H}\mathbf{R}\mathbf{H}^\top\mathbf{H}\mathbf{z}_\nu,\end{aligned}$$

since $\boldsymbol{\nu} = \mathbf{z}_\nu + \boldsymbol{\alpha}$ and $\mathbf{H}^\top\mathbf{H} = \mathbf{I}_{3\times3}$ (identity matrix). Defining $\mathbf{H}\mathbf{R}\mathbf{H}^\top = \mathbf{R}_\mathrm{B}$, i.e.,

$$\mathbf{R}_\mathrm{B} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}, \quad (25)$$

leads to

$$\dot{V}_\varepsilon = \boldsymbol{\varepsilon}^\top\left(\mathbf{R}_\mathrm{C}^\top\mathbf{R}_\mathrm{B}\mathbf{H}\boldsymbol{\alpha} - \mathbf{v}_\mathrm{c}\right) + \boldsymbol{\varepsilon}^\top\mathbf{R}_\mathrm{C}^\top\mathbf{R}_\mathrm{B}\mathbf{H}\mathbf{z}_\nu,$$

where $\mathbf{R}_\mathrm{B}\mathbf{H}\boldsymbol{\alpha} = \mathbf{R}_\mathrm{B}\boldsymbol{\alpha}_{\mathrm{v,B}}$ with $\boldsymbol{\alpha}_{\mathrm{v,B}} = \boldsymbol{\alpha}_\mathrm{v} = [\alpha_\mathrm{u}, \alpha_\mathrm{v}]^\top$. Now, $\mathbf{R}_\mathrm{B}\boldsymbol{\alpha}_{\mathrm{v,B}} = \mathbf{R}_\mathrm{DV}\boldsymbol{\alpha}_{\mathrm{v,DV}} = \boldsymbol{\alpha}_{\mathrm{v,I}}$ represents the desired linear velocity decomposed in **I**, while $\boldsymbol{\alpha}_{\mathrm{v,DV}} = [U_\mathrm{d}, 0]^\top$, where $U_\mathrm{d} = |\boldsymbol{\alpha}_\mathrm{v}|$, represents the desired linear velocity decomposed in a DESIRED VELOCITY frame (**DV**). This frame is aligned with $\boldsymbol{\alpha}_{\mathrm{v,I}}$, whose orientation we want to design so as to achieve path following. Hence, we get

$$\begin{aligned}\dot{V}_\varepsilon &= \boldsymbol{\varepsilon}^\top\left(\mathbf{R}_\mathrm{C}^\top\mathbf{R}_\mathrm{DV}\boldsymbol{\alpha}_{\mathrm{v,DV}} - \mathbf{v}_\mathrm{c}\right) + \boldsymbol{\varepsilon}^\top\mathbf{R}_\mathrm{C}^\top\mathbf{R}_\mathrm{B}\mathbf{H}\mathbf{z}_\nu \\ &= \boldsymbol{\varepsilon}^\top\left(\mathbf{R}_\mathrm{R}\boldsymbol{\alpha}_{\mathrm{v,DV}} - \mathbf{v}_\mathrm{c}\right) + \boldsymbol{\varepsilon}^\top\mathbf{R}_\mathrm{C}^\top\mathbf{R}_\mathrm{B}\mathbf{H}\mathbf{z}_\nu,\end{aligned}$$

where $\mathbf{R}_\mathrm{C}^\top(\chi_\mathrm{c})\mathbf{R}_\mathrm{DV}(\chi_\mathrm{d}) = \mathbf{R}_\mathrm{R}(\chi_\mathrm{d} - \chi_\mathrm{c})$, i.e., the relative orientation between **C** and **DV**.

Thus, U_c and $(\chi_\mathrm{d} - \chi_\mathrm{c})$ can be considered as virtual inputs for driving $\boldsymbol{\varepsilon}$ to zero, given that $U_\mathrm{d} > 0$. Denote the angular difference by $\chi_\mathrm{r} = \chi_\mathrm{d} - \chi_\mathrm{c}$, and expand the CLF derivative to obtain

$$\dot{V}_\varepsilon = s(U_\mathrm{d}\cos\chi_\mathrm{r} - U_\mathrm{c}) + eU_\mathrm{d}\sin\chi_\mathrm{r} + \boldsymbol{\varepsilon}^\top\mathbf{R}_\mathrm{C}^\top\mathbf{R}_\mathrm{B}\mathbf{H}\mathbf{z}_\nu,$$

where U_c can be chosen as

$$U_\mathrm{c} = U_\mathrm{d}\cos\chi_\mathrm{r} + \gamma s \quad (26)$$

with $\gamma > 0$ constant, while χ_r can be selected as

$$\chi_\mathrm{r} = \arctan\left(-\frac{e}{\triangle_\mathrm{e}}\right) \quad (27)$$

with $\triangle_\mathrm{e} > 0$ (not necessarily constant), giving

$$\dot{V}_\varepsilon = -\gamma s^2 - U_\mathrm{d}\frac{e^2}{\sqrt{e^2 + \triangle_\mathrm{e}^2}} + \boldsymbol{\varepsilon}^\top\mathbf{R}_\mathrm{C}^\top\mathbf{R}_\mathrm{B}\mathbf{H}\mathbf{z}_\nu. \quad (28)$$

Consequently

$$\dot{\varpi}_{\mathrm{c}} = \frac{U_{\mathrm{c}}}{|\mathbf{p}'_{\mathrm{c}}|} \tag{29}$$

and

$$\chi_{\mathrm{d}} = \chi_{\mathrm{c}} + \chi_{\mathrm{r}}, \tag{30}$$

with U_{c} as in (26), χ_{c} as in (22), and χ_{r} as in (27). Equations (29) and (30) indicate that the collaborator point continuously leads the ship, while the desired linear velocity of the ship must point at the collaborator's path-tangential, in the direction of forward motion. Specifically, the desired linear velocity $\boldsymbol{\alpha}_{\mathrm{v}}$ must be computed by

$$\begin{aligned}\boldsymbol{\alpha}_{\mathrm{v}} &= \mathbf{R}_{\mathrm{B}}^{\top}\mathbf{R}_{\mathrm{DV}}\boldsymbol{\alpha}_{\mathrm{v,DV}} \\ &= \begin{bmatrix} U_{\mathrm{d}}\cos(\chi_{\mathrm{d}} - \psi) \\ U_{\mathrm{d}}\sin(\chi_{\mathrm{d}} - \psi) \end{bmatrix}. \end{aligned} \tag{31}$$

Then, write the system dynamics of $\boldsymbol{\varepsilon}$ and $\mathbf{z}_{\mathrm{g}}$ as

$$\Sigma_1 : \dot{\boldsymbol{\varepsilon}} = \mathbf{f}_1(t, \boldsymbol{\varepsilon}) + \mathbf{g}_1(t, \boldsymbol{\varepsilon}, \mathbf{z}_{\mathrm{g}})\mathbf{z}_{\mathrm{g}} \tag{32}$$

$$\Sigma_2 : \dot{\mathbf{z}}_{\mathrm{g}} = \mathbf{f}_2(t, \mathbf{z}_{\mathrm{g}}), \tag{33}$$

which is a pure cascade where the control subsystem perturbs the guidance subsystem through the interconnection matrix

$$\mathbf{g}_1(t, \boldsymbol{\varepsilon}, \mathbf{z}_{\mathrm{g}}) = \left[\mathbf{0}_{2\times 1}, \mathbf{R}_{\mathrm{C}}^{\top}\mathbf{R}_{\mathrm{B}}\mathbf{H}\right]. \tag{34}$$

Also, consider the following assumptions

Assumption 3. $|\mathbf{p}'_{\mathrm{p}}| \in \left[|\mathbf{p}'_{\mathrm{p}}|_{\min}, |\mathbf{p}'_{\mathrm{p}}|_{\max}\right] \; \forall \varpi \in \mathbb{R}$

Assumption 4. $\triangle_{\mathrm{e}} \in [\triangle_{\mathrm{e,\,min}}, \infty\rangle$, $\triangle_{\mathrm{e,\,min}} > 0$

Assumption 5. $U_{\mathrm{d}} \in [U_{\mathrm{d,min}}, \infty\rangle$, $U_{\mathrm{d,min}} > 0$

By considering the state vector $\boldsymbol{\xi} = \left[\boldsymbol{\varepsilon}^{\top}, \mathbf{z}_{\mathrm{g}}^{\top}\right]^{\top}$, we arrive at the following proposition

Proposition 6. The equilibrium point $\boldsymbol{\xi} = \mathbf{0}$ is rendered uniformly globally asymptotically and locally exponentially stable (UGAS/ULES) under assumptions (3-5) when applying (18) with the reference signals (17) and (31).

PROOF. Since the origin of system Σ_2 is shown to be UGES in Proposition 2, the origin of the unperturbed system Σ_1 (i.e., when $\mathbf{z}_{\mathrm{g}} = \mathbf{0}$) is trivially shown to be UGAS/ULES by applying standard Lyapunov theory to (20) and (28) (actually, it is uniformly semi-globally exponentially stable, i.e., USGES), and the interconnection term satisfies $\left|\mathbf{g}_1(t, \boldsymbol{\varepsilon}, \mathbf{z}_{\mathrm{g}})\right| = 1$, the proposed result follows directly from Theorem 7 and Lemma 8 of (Panteley *et al.* 1998).

This result means that path following is achieved globally, uniformly in time. Note that the established stability property of Proposition 6 also is known as global κ-exponential stability (Sørdalen and Egeland 1995). Finally, note that by choosing

$$U_{\mathrm{d}} = \kappa\sqrt{e^2 + \triangle_{\mathrm{e}}^2}, \tag{35}$$

where $\kappa > 0$, the origin of $\boldsymbol{\xi}$ can be shown to be UGES. Although very powerful, such a result is clearly not physically achievable due to natural speed limitations. Consequently, Proposition 6 states the best possible stability property achievable for any vehicle.

2.4.3. Step 3: Synchronization Loop Design In this final design step, we determine the required *size* of $\boldsymbol{\alpha}_{\mathrm{v}}$, i.e., U_{d}, such that a ship controlled by (18) with reference signals given by (17) and (31) achieves synchronization with the target point. This synchronization loop design is inspired by the formation control approach of (Breivik *et al.* 2006). Consequently, consider the positive definite and radially unbounded CLF

$$V_{\tilde{\varpi}} = \frac{1}{2}\tilde{\varpi}^2, \tag{36}$$

where

$$\tilde{\varpi} = \varpi_{\mathrm{c}} - \varpi_{\mathrm{t}}, \tag{37}$$

and differentiate the CLF with respect to time to get

$$\begin{aligned}\dot{V}_{\tilde{\varpi}} &= \tilde{\varpi}\dot{\tilde{\varpi}} \\ &= \tilde{\varpi}(\dot{\varpi}_{\mathrm{c}} - \dot{\varpi}_{\mathrm{t}}) \\ &= \tilde{\varpi}(z_{\mathrm{c}} + \alpha_{\mathrm{c}} - \dot{\varpi}_{\mathrm{t}})\end{aligned}$$

where $z_{\mathrm{c}} = \dot{\varpi}_{\mathrm{c}} - \alpha_{\mathrm{c}}$, and α_{c} represents the desired speed of the collaborator when the ship has converged to the path, i.e., when $\boldsymbol{\xi} = \mathbf{0}$. Hence, we have that

$$\dot{V}_{\tilde{\varpi}} = \tilde{\varpi}\left(\frac{U_{\mathrm{d}}}{|\mathbf{p}'_{\mathrm{c}}|} - \frac{U_{\mathrm{t}}}{|\mathbf{p}'_{\mathrm{t}}|}\right) + \tilde{\varpi}z_{\mathrm{c}},$$

which is equal to

$$\dot{V}_{\tilde{\varpi}} = -k_{\tilde{\varpi}}\frac{\tilde{\varpi}^2}{\sqrt{\tilde{\varpi}^2 + \triangle_{\tilde{\varpi}}^2}} + \tilde{\varpi}z_{\mathrm{c}} \tag{38}$$

when choosing

$$U_{\mathrm{d}} = |\mathbf{p}'_{\mathrm{c}}|\left(\frac{U_{\mathrm{t}}}{|\mathbf{p}'_{\mathrm{t}}|} - k_{\tilde{\varpi}}\frac{\tilde{\varpi}}{\sqrt{\tilde{\varpi}^2 + \triangle_{\tilde{\varpi}}^2}}\right), \tag{39}$$

where $\triangle_{\tilde{\varpi}} \in [\triangle_{\tilde{\varpi},\,\min}, \infty\rangle$, $\triangle_{\tilde{\varpi},\,\min} > 0$, and where

$$k_{\tilde{\varpi}} = \sigma\frac{U_{\mathrm{t}}}{|\mathbf{p}'_{\mathrm{t}}|}, \sigma \in \langle 0, 1] \tag{40}$$

ensures that U_{d} satisfies Assumption 5 by leading to

$$U_{\mathrm{d}} = U_{\mathrm{t}}\left(1 - \sigma\frac{\tilde{\varpi}}{\sqrt{\tilde{\varpi}^2 + \triangle_{\tilde{\varpi}}^2}}\right)\frac{|\mathbf{p}'_{\mathrm{c}}|}{|\mathbf{p}'_{\mathrm{t}}|}. \tag{41}$$

Then, expand z_{c} to get

$$\begin{aligned}z_{\mathrm{c}} &= \dot{\varpi}_{\mathrm{c}} - \alpha_{\mathrm{c}} \\ &= \frac{U_{\mathrm{d}}(\cos\chi_{\mathrm{r}} - 1) + \gamma s}{|\mathbf{p}'_{\mathrm{c}}|}\end{aligned} \tag{42}$$

where

$$(\cos\chi_{\mathrm{r}} - 1) = \frac{\triangle_{\mathrm{e}} - \sqrt{e^2 + \triangle_{\mathrm{e}}^2}}{\sqrt{e^2 + \triangle_{\mathrm{e}}^2}} \tag{43}$$

such that the system dynamics of $\tilde{\varpi}$ and $\boldsymbol{\xi}$ can be written as

$$\Sigma_3 : \dot{\tilde{\varpi}} = f_3(t, \tilde{\varpi}) + \mathbf{g}_3(t, \tilde{\varpi}, \boldsymbol{\xi})^\top \boldsymbol{\xi} \tag{44}$$

$$\Sigma_4 : \dot{\boldsymbol{\xi}} = \mathbf{f}_4(t, \boldsymbol{\xi}), \tag{45}$$

which we recognize as a cascade where the control and guidance subsystems perturb the synchronization subsystem through the interconnection vector

$$\mathbf{g}_3(t, \tilde{\varpi}, \boldsymbol{\xi}) = \frac{1}{|\mathbf{p}_c'|} \left[\gamma, U_d \frac{\triangle_e - \sqrt{e^2 + \triangle_e^2}}{e\sqrt{e^2 + \triangle_e^2}}, \mathbf{0}_{1\times 4} \right]^\top, \tag{46}$$

which is well-defined since

$$\lim_{e \to 0} \frac{\triangle_e - \sqrt{e^2 + \triangle_e^2}}{e\sqrt{e^2 + \triangle_e^2}} = 0. \tag{47}$$

Note that the cascade is pure in the sense that the control subsystem excites the guidance subsystem, which in turn excites the synchronization subsystem.

Considering the state vector $\boldsymbol{\zeta} = \left[\tilde{\varpi}, \boldsymbol{\xi}^\top\right]^\top$, the following theorem can now be stated

Theorem 7. (Guided Motion Control). The equilibrium point $\boldsymbol{\zeta} = \mathbf{0}$ is rendered UGAS/ULES under assumptions (3-4) when applying (18) with reference signals (17) and (31) employing (41).

PROOF. Since the origin of system Σ_4 is shown to be UGAS/ULES in Proposition 6, the origin of the unperturbed system Σ_3 (i.e., when $\boldsymbol{\xi} = \mathbf{0}$) is trivially shown to be UGAS/ULES by applying standard Lyapunov theory to (36) and (38), and the interconnection term satisfies $|\mathbf{g}_3(t, \tilde{\varpi}, \boldsymbol{\xi})| < |\mathbf{p}_p'|_{\min}^{-1} \left(\gamma^2 + \left(\frac{U_{t,\max}}{\triangle_{e,\min}} \right)^2 \right)^{1/2}$, the proposed result follows directly from Theorem 7 and Lemma 8 of (Panteley *et al.* 1998).

Consequently, the trajectory tracking problem as stated in (6) has been solved in three design steps.

3. DISCUSSION

What can be gained by the more complex and involved control design and analysis associated with the guided approach? A very important aspect is that the guided scheme readily extends to underactuated ships since it redefines the output space to the linear velocity and heading, where the heading either can be controlled independently (fully actuated case) or must be dedicated to control the direction of the linear velocity (underactuated case). Hence, leaving the control subsystem unchanged, a basic redesign of the guidance and synchronization subsystems is all that is required. In fact, for straight-line paths no redesign is necessary if the desired heading is assigned as the desired orientation of the linear velocity. Such extendability also suggests a unified motion controller capable of handling a ship seamlessly between its actuation regimes. Furthermore, the servoed approach does not exploit any available path information, by only considering the target point. A consequence is degraded transient convergence behavior, which becomes erratic and unnatural. This aspect contrasts starkly with the guided approach, where a collaborator point guides the ship gently toward the path. In real life, this difference affects wear and tear of the actuators, as well as fuel consumption. Additionally, the commanded control input of the servoed scheme grows unbounded with the ship-target error, rendering the ship-path system unstable if the ship cannot keep up with the target. This problem does not affect the guided approach, whose commanded control input saturates at a tunable threshold, rendering the ship-path system stable even though the ship-target system may become unstable. Such a path following feature distinguishes the guided approach, and probably extends well to real-life demands of limited propulsion capacity. The stability properties of Theorems 1 and 7 also reflect this reasoning, where the former looks more impressive, but where the latter represent the physically attainable alternative.

4. CONCLUSIONS

This paper treated the topic of motion control for fully actuated ships in a trajectory tracking scenario by developing and discussing two fundamentally different motion control concepts. The servoed concept corresponds to standard DP controllers found in the established ship control literature, while the guided concept represents a novel scheme inspired by research within missile guidance, path following, and formation control.

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