CLARIFICATION OF THE LOW-FREQUENCY MODELLING CONCEPT FOR MARINE CRAFT

Andrew Ross * **Tristan Perez** * **Thor I. Fossen** *,**

* *Centre for Ships and Ocean Structures*
Norwegian University of Science and Technology
NO-7491 Trondheim, Norway
** *Department of Engineering Cybernetics*
Norwegian University of Science and Technology
NO-7491 Trondheim, Norway
E-mail: andrew.ross@marin.ntnu.no,
tristan.perez@marin.ntnu.no, fossen@ieee.org

Abstract: This article presents some remarks on models currently used in low speed manoeuvring and dynamic positioning problems. It discusses the relationship between the classical hydrodynamic equations for manoeuvring and seakeeping, and offers insight into the models used for simulation and control system design.

Keywords: manoeuvring, dynamic positioning, hydrodynamics

INTRODUCTION

The aim of this paper is to discuss some issues associated with the physical motivation behind models used for low speed manoeuvring and dynamic positioning. Such a model should be suitable for control or simulation purposes, and well-founded in hydrodynamic theory.

The description of the forces acting on a ship have been developed from two different schools of thought, with different motivations: *manoeuvring theory* and *seakeeping theory*. In manoeuvring theory, the ship is assumed to act without the presence of waves, and the model is generally represented in terms of the hydrodynamic derivatives. Seakeeping theory is motivated by the desire to understand how a ship behaves in response to the waves encountered in a typical seaway, and to judge these responses in terms of operability and safety requirements. Much work has focused on combining these two fields, in order to make the practice of having one low-frequency model and one wave-frequency model superfluous.

The unification of these two distinct but related fields began with the work of Cummins (1962), in which a time-domain model no longer reliant on sinusoidal motion was derived. This formulation was related to the classical frequency domain approach by Ogilvie (1964). Using this as the framework, Bailey *et al.* (1997) formulated a model which captured the behaviour of both. State-space solutions to the integro-differential equations were derived, for example in Kristiansen and Egeland (2003). Fossen (2005) built upon the work by Bailey et. al. and Kristiansen, and introduced a framework in which the results from hydrodynamics software could be incorporated seamlessly, resulting in a state-space model that is accurate for manoeuvring, station-keeping and control in a seaway. An approximate model for manoeuvring should capture as much of the accuracy of this approach as is practicable, but simplify it such that it is formed through constant coefficients, making it simple and easy to apply.

At present, the coefficients of a manoeuvring model are generally taken at zero frequency values, known as the hydrodynamic derivatives. This is unsatisfactory from a hydrodynamic point of view, especially for roll, pitch and heave.

This paper proceeds by laying out the classical frequency domain approach, then describing its equivalent time-domain description, known as the Cummins equation. The typical manoeuvring model is then presented, with analysis of the zero-frequency hydrodynamic derivatives, with especial regard to their relationship with the Cummins equation. A more suitable approximation for use as a manoeuvring model is then presented, with two symbolic and numerical examples.

1. CLASSICAL MODEL

From Faltinsen (1990) and Journée and Massie (2001), the classical frequency-dependent hydrodynamic model can be written:

$$(\mathbf{M}_{RB}+\mathbf{A}(\omega))\ddot{\boldsymbol{\xi}}+\mathbf{B}(\omega)\dot{\boldsymbol{\xi}}+\mathbf{C}\boldsymbol{\xi}=\mathbf{f}_{FK+diff}+\mathbf{f} \quad (1)$$

Where $\omega\in\mathbb{R}^+$ is the frequency of encounter, $\mathbf{M}_{RB}\in\mathbb{R}^{6\times6}$ is the rigid body added mass, $\mathbf{A}(\omega)\in\mathbb{R}^{6\times6}$ is the added mass as a function of frequency, $\mathbf{B}(\omega)\in\mathbb{R}^{6\times6}$ is the linear damping matrix, $\mathbf{C}\in\mathbb{R}^{6\times6}$ is the matrix of linear restoring force coefficients, $\boldsymbol{\xi}\in\mathbb{R}^6$ is the position vector relative to the $\{h\}$-frame: a frame located at the mean position of the vessel, $\mathbf{f}_{FK+diff}\in\mathbb{R}^6$ is the vector of wave induced forces (Froude-Krylov and diffraction), and finally $\mathbf{f}\in\mathbb{R}^6$ is the vector of excitation forces.

Implicitly, this equation only allows oscillatory behaviour at a single frequency, although the various frequencies can be easily superposed. It is only a pseudo-differential equation, satisfying the properties of an ODE only when the displacement $\boldsymbol{\xi}$, its derivatives, and the excitation forces are purely sinusoidal at frequency ω. That is, there can be no transient motion, or excitation: if there were, the equation itself would have no meaning. The classical model was never envisaged to be able to cope with anything other than steady state, oscillatory motion. So long as the response is purely sinusoidal, (1) maintains the illusion of being a conventional differential equation.

2. THE CUMMINS EQUATION

Consider Newton's law for 6 DOF motions in the form (neglecting viscous effects):

$$\mathbf{M}_{RB}\ddot{\boldsymbol{\xi}}+\mathbf{C}\boldsymbol{\xi}=\mathbf{f}_{rad}+\mathbf{f}_{FK+diff}+\mathbf{f} \quad (2)$$

where $\mathbf{f}_{rad}\in\mathbb{R}^6$ represents the generalized radiation forces. This is based on the use of potential theory: there are no viscous forces in the fluid. However, viscous effects can be included separately under the principle of linear superposition. From Cummins (1962), the time domain solution for the generalized hydrodynamic radiation force, $\mathbf{f}_{rad}$, at zero forward speed can be written:

$$\mathbf{f}_{rad}=\bar{\mathbf{A}}\ddot{\boldsymbol{\xi}}+\int_{-\infty}^{t}\mathbf{K}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)\,d\tau \quad (3)$$

where $\bar{\mathbf{A}}\in\mathbb{R}^{6\times6}$ is a constant added-mass matrix to be determined later, and $\mathbf{K}(t)\in\mathbb{R}^{6\times6}\times\mathbb{R}^+$ is a matrix function of impulse response functions, which are as yet unknown. For a causal system, this reduces to:

$$\mathbf{f}_{rad}=\bar{\mathbf{A}}\ddot{\boldsymbol{\xi}}+\int_{0}^{t}\mathbf{K}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)\,d\tau \quad (4)$$

$$\equiv\bar{\mathbf{A}}\ddot{\boldsymbol{\xi}}+\int_{0}^{t}\mathbf{K}(\tau)\dot{\boldsymbol{\xi}}(t-\tau)\,d\tau \quad (5)$$

Equations (4) and (5) are extremely valuable, since they have the capability to describe the forces regardless of the frequency of oscillation: the equations hold during sinusoidal, transient or even random motion. Additionally, there are no limitations placed on the forcing function. This not only means that non-sinusoidal excitations are possible, but also that any nonlinear forces, such as nonlinear damping, wind, or other environmental loads, can be included on the right hand side of the Cummins equation as nonlinear excitation forces. The equations of motion from (2) can then be written as:

$$\left(\mathbf{M}_{RB}+\bar{\mathbf{A}}\right)\ddot{\xi}+\int_{0}^{t}\mathbf{K}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)\,d\tau+\mathbf{C}\boldsymbol{\xi}=\mathbf{f}_{FK+diff} \quad (6)$$

There is another important feature of these equations. The terms $\bar{\mathbf{A}}$ and $\mathbf{K}(t)$ are functions *only* of the geometry of the vessel: they do not vary with the motion of the vessel. This is in sharp contrast to the classical model (1), in which $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ vary with the frequency of oscillation and thus with the motion of the ship. This in fact bears a great deal of similarity to the task of finding a manoeuvring model, in which the coefficients are invariant with respect to the velocity of the ship.

2.1 Relationship between the Cummins Equation and the Classical Model

The Cummins equation is extremely useful, since it can describe behaviour under an arbitrary forcing function, and for any and all frequencies simultaneously. This is, of course, presuming that the kernels of the convolution integrals are known. Unfortunately, this is generally not the case. Take Cummins equation in the form:

$$(\mathbf{M}_{RB}+\bar{\mathbf{A}})\,\ddot{\boldsymbol{\xi}}+\int_0^t \mathbf{K}(\tau)\,\dot{\boldsymbol{\xi}}(t-\tau)\,d\tau + \mathbf{C}\boldsymbol{\xi} = \mathbf{f}_{FK+diff} + \mathbf{f} \quad (7)$$

Ogilvie (1964) showed the relationship between the classical approach (1) and the time domain approach (7), and therefore showed how the results from the former could be applied to solve the latter. This is done by assuming sinusoidal motion, and separating the convolution integral into two parts: one in phase with velocity, and the other in phase with acceleration. By doing this, it is easy to show that the following equations describe the relationship between the frequency domain and time domain approaches:

$$\mathbf{A}(\omega) = \bar{\mathbf{A}} - \lim_{t\to\infty}\frac{1}{\omega}\int_0^t \mathbf{K}(\tau)\sin(\omega\tau)\,d\tau \quad (8)$$

$$\mathbf{B}(\omega) = \lim_{t\to\infty}\int_0^t \mathbf{K}(\tau)\cos(\omega\tau)\,d\tau \quad (9)$$

To solve for $\bar{\mathbf{A}}$, we can evaluate equation (8) at $\omega = \infty$, and apply the *Riemann-Lebesgue lemma* giving $\bar{\mathbf{A}} = \mathbf{A}(\infty)$. Additionally, it is fairly easy to see that $\mathbf{K}(t)$ must be related to the sine and cosine Fourier-transforms of the frequency response functions, and that it can be calculated for example by:

$$\mathbf{K}(t) = \frac{2}{\pi}\int_0^\infty \mathbf{B}(\omega)\cos(\omega t)\,d\omega \quad (10)$$

Using these relations, the equation of motion is:

$$(\mathbf{M}_{RB} + \mathbf{A}(\infty))\,\ddot{\boldsymbol{\xi}} + \left(\mathbf{K} * \dot{\boldsymbol{\xi}}\right)(t) + \mathbf{C}\boldsymbol{\xi} = \mathbf{f}_{FK+diff} + \mathbf{f} \quad (11)$$

The matrix $\mathbf{A}(\infty)$ can be interpreted in a physical sense, and is certainly not just a mathematical artifice. Essentially, it explains the instantaneous reaction of the fluid to any motion. As soon as movement begins, the $\mathbf{A}(\infty)$ term explains the inertial reaction of the fluid to this sudden motion. After the instantaneous reaction, the remainder of the reaction is described by the memory effect, through the convolution integral.

2.2 Seakeeping Model

The equations of motion (11) are recognised as the seakeeping model:

$$(\mathbf{M}_{RB}+\mathbf{A}(\infty))\,\ddot{\boldsymbol{\xi}} + \mathbf{B}_v\dot{\boldsymbol{\xi}} + \boldsymbol{\mu} + \mathbf{C}\boldsymbol{\xi} = \mathbf{f}_{FK+diff} + \mathbf{f} \quad (12)$$

where the additional matrix $\mathbf{B}_v \in \mathbb{R}^{6\times6}$ represents linear viscous damping added directly in the time domain (Ross and Fossen 2005), and where potential damping $\mathbf{B}(\omega)$ gives rise to the term:

$$\boldsymbol{\mu} = \int_0^t \mathbf{K}(t-\tau)\,\dot{\boldsymbol{\xi}}(\tau)\,d\tau = \int_0^t \mathbf{K}(\tau)\,\dot{\boldsymbol{\xi}}(t-\tau)\,d\tau \quad (13)$$

which is the hydrodynamic force due to fluid memory effects in an ideal fluid. It is convenient to represent $\boldsymbol{\mu}$ as a state-space model (Kristiansen and Egeland 2003, Fossen 2005):

$$\dot{\boldsymbol{\chi}} = \mathbf{A}_s\boldsymbol{\chi} + \mathbf{B}_s\dot{\boldsymbol{\xi}} \quad (14)$$

$$\boldsymbol{\mu} = \mathbf{C}_s\boldsymbol{\chi} + \mathbf{D}_s\dot{\boldsymbol{\xi}} \quad (15)$$

where this SS model approximates (13): the impulse response convolved with velocity. It is also possible to add frequency-dependent viscous damping instead of the constant matrix $\mathbf{B}_v$. If this is done, the impulse response functions $\mathbf{K}(t)$ should include this effect as well.

2.3 Manoeuvring Model

For manoeuvring, it is common to represent the vessel models in (Fossen 1994, Fossen 2002) in the {b}-frame as:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\theta)\boldsymbol{\nu}$$

$$(\mathbf{M}_{RB}+\mathbf{M}_A)\,\dot{\boldsymbol{\nu}} + \mathbf{D}(\boldsymbol{\nu})\,\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}_{FK+diff} + \boldsymbol{\tau}$$

where $\mathbf{M}_A \in \mathbb{R}^{6\times6}$ is an added mass matrix, $\mathbf{D}(\boldsymbol{\nu}) \in \mathbb{R}^{6\times6}$ is a matrix of damping coefficients, and $\mathbf{g}(\boldsymbol{\eta}) \in \mathbb{R}^6$ is a vector of restoring forces. This model uses frequency independent parameters.

2.4 The Relationship Between the Seakeeping and Maneuvering Models

The kinematic transformation matrix $\mathbf{J}^*(\boldsymbol{\theta}^*) \in \mathbb{R}^{6\times6}$ between the seakeeping {h}-frame and the vessel {b}-frame satisfies (Fossen 2005):

$$\dot{\boldsymbol{\xi}} = \mathbf{J}^*(\boldsymbol{\theta}^*)\boldsymbol{\nu} \approx \mathbf{J}^*\boldsymbol{\nu}$$

for small angles $\boldsymbol{\theta}^* = [\xi_4, \xi_5, \xi_6]^\top$. Hence, we have assumed that the matrices $\mathbf{M}_{RB}, \mathbf{A}$,and $\mathbf{B}_v$ in (12) are independent of the rotations $\boldsymbol{\theta}^*$ of the body fixed {b}-frame with respect to the {h}-frame and it follows that $\mathbf{A} = \mathbf{J}^{*\top}\mathbf{A}\mathbf{J}^*$ and so on. Assume that the hydrodynamic data $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ are computed in CG and that the {b}-frame origin is located at the CG such that $\mathbf{J}^* = \mathbf{I}$. Consequently, the seakeeping and manoeuvring models in Sections 2.2 and 2.3 are equivalent representations if we choose:

$$\mathbf{M}_A = \mathbf{A}(\infty)$$

$$\mathbf{D}(\boldsymbol{\nu}) = \mathbf{B}_v + \mathbf{B}_p(s)$$

$$\mathbf{g}(\boldsymbol{\eta}) = \mathbf{C}\boldsymbol{\eta}$$

$$\boldsymbol{\tau}_{FK+diff} = \mathbf{f}_{FK+diff}$$

$$\boldsymbol{\tau} = \mathbf{f}$$

where $\mathbf{B}_p(s)$ is a matrix of transfer functions describing the potential damping, each of which comes from

(14) and (15), i.e. $\mathbf{B}_p(s) = \mathbf{D}_s + \mathbf{C}_s(s\mathbf{I} - \mathbf{A}_s \mathbf{s})^{-1}\mathbf{B}_s$, and $\boldsymbol{\mu} = \mathbf{B}_p(s)\dot{\boldsymbol{\xi}}$. This choice for $\mathbf{M}_A, \mathbf{D}(\boldsymbol{\nu})$, and $\mathbf{g}(\boldsymbol{\eta})$ is referred to as the *unified model* by Fossen (2005) and it deviates from the *zero frequency approach* which is used in classical manoeuvring theory [1]. This is the topic for the next section.

3. MANOEUVRING MODEL

A low speed manoeuvring model is one which is suitable to describe the motion of a ship in calm water. It is frequency invariant, and kept as basic as possible so that it may be used for quick analysis, simulation and control. At present, there is a sizeable disconnect between a physically motivated model, and one that is useful for control and simulation purposes.

Unfortunately, it is not possible to properly decouple the coefficients of the model from the motion itself. The motion of the ship is not known a priori in a manoeuvring or dynamic positioning problem, and since the coefficients themselves are inherently functions of the motion of the vessel, it is quite awkward to choose one set of coefficients that is useful for any task. Nonetheless, keeping this limitation in mind, the model can be written in the form (Fossen 1994, Fossen 2002):

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\theta})\boldsymbol{\nu} \quad (16)$$

$$(\mathbf{M}_{RB} + \mathbf{M}_A)\dot{\boldsymbol{\nu}} + \mathbf{D}\boldsymbol{\nu} + \mathbf{d}_n(\boldsymbol{\nu}) + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau}_{FK+diff} + \boldsymbol{\tau} \quad (17)$$

For zero-speed, we have the simple relationships:

$$\tilde{\mathbf{M}}_A(\omega) = \mathbf{A}(\omega) \quad (18)$$

$$\tilde{\mathbf{D}}(\omega) = \mathbf{B}(\omega) + \mathbf{B}_v(\omega) \quad (19)$$

where we have added a viscous damping term $\mathbf{B}_v(\omega)$. This term could correspond to exponential ramps in DOF 1, 2 and 6, as in Ross and Fossen (2005). In roll, it is common to use the model of Ikeda *et al.* (1976). It is advantageous to use a model with frequency-independent matrices when designing control systems. We will now discuss how a low-frequency (LF) model for this can be designed.

Classical LF (Zero-Frequency) Model

The most common way to choose the constant matrices $\mathbf{M}_A$ and $\mathbf{D}$ in a control plant model is to use LF added mass and damping terms:

$$\mathbf{M}_A = \lim_{\omega \to 0} \mathbf{A}(\omega) \quad (20)$$

$$\mathbf{D} = \lim_{\omega \to 0} (\mathbf{B}(\omega) + \mathbf{B}_v(\omega)) \quad (21)$$

[1] Note that $\mathbf{D}(\boldsymbol{\nu})$ is modelled as a constant matrix, $\mathbf{B}_v$, plus a matrix of transfer functions $\mathbf{B}_p(s)$

The linear viscous damping can be modelled as a diagonal matrix:

$$\mathbf{B}_v(\omega) = \begin{bmatrix} \beta_1 e^{-\alpha\omega} & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_2 e^{-\alpha\omega} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_{\text{IKEDA}}(\omega) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_6 e^{-\alpha\omega} \end{bmatrix}$$

where $\alpha > 0$ is the exponential rate, $\beta_{1,2,6} > 0$ are the zero frequency viscous damping coefficients, and $B_{\text{IKEDA}}(\omega)$ is frequency-dependent roll damping function based on the theory of Ikeda *et al.* (1976) for instance. Viscous damping in the other modes are assumed to be negligible. However, these terms can easily be included if necessary. Bailey *et al.* (1997) described the relationship between the hydrodynamic derivatives and the Cummins equation. This was done by taking the limits of equations (8) and (9) as $\omega \to 0$. This relationship is described in the following integrals:

$$\mathbf{M}_A = \lim_{\omega \to 0} \mathbf{A}(\omega) = \mathbf{A}(\infty) - \lim_{\omega \to 0} \int_0^\infty \mathbf{K}(\tau) \frac{\sin(\omega\tau)}{\omega} d\tau = \mathbf{A}(\infty) - \int_0^\infty \tau \mathbf{K}(\tau) d\tau \quad (22)$$

$$\mathbf{D} = \lim_{\omega \to 0} (\mathbf{B}(\omega) + \mathbf{B}_v(\omega)) = \lim_{\omega \to 0} \int_0^\infty (\mathbf{K}(\tau) + \mathbf{K}_v(\tau)) \cos(\omega\tau) d\tau = \int_0^\infty \mathbf{K}(\tau) d\tau + \int_0^\infty \mathbf{K}_v(\tau) d\tau \quad (23)$$

Notice that the hydrodynamic derivatives are given by the area under the impulse response function due to potential and viscous damping. The impulse response function due to viscous damping is denoted $\mathbf{K}_v(t)$. The hydrodynamic derivatives in $\mathbf{M}_A$ is the negative moment of the area under the impulse response function plus the added mass at infinite frequency. The hydrodynamic derivatives are appropriate for surge, sway and yaw, in which there is little oscillatory motion during manouvering, that is $\omega \approx 0$. However, they are not applicable for roll, pitch or heave, in which motion tends to be at a natural frequency $\omega_0 > 0$.

3.1 Redefined LF Model

The classical low frequency model isn't a good approximation for roll, pitch and heave. Indeed, it is unclear what zero-frequency would entail in these degrees of freedom.

Definition 1 (LF Model) The LF model is frequency independent, and is valid for dynamic positioning and manoeuvring. It consists of the following sub-models:

1) Zero-Frequency model for surge sway and yaw (DOF 1,2,6): These DOF consist of mass-damper systems.

2) Natural frequency model for heave, roll and pitch (DOF 3,4,5): In these DOF, we have mass-spring-damper systems.

It is appropriate to use the relations shown in equations (8)–(9), where the retardation functions are replaced with constant parameters. We will approximate Cummins equation at $\omega = 0$ for DOF 1,2, and 6, and for DOF 3, 4 and 5,at $\omega = \omega_z, \omega_\phi, \omega_\theta$: the natural frequencies, where the subscripts denote heave, roll and pitch respectively.

From (22)–(23) we obtain:

$$\mathbf{M}_A^{1,2,6} = \lim_{\omega \to 0} \mathbf{A}^{1,2,6}(\omega) = \mathbf{A}(\infty) - \int_0^\infty \tau \mathbf{K}(\tau)\, d\tau$$

$$\mathbf{D}^{1,2,6} = \lim_{\omega \to 0} \left(\mathbf{B}^{1,2,6}(\omega) + \mathbf{B}_v^{1,2,6}(\omega)\right) = \int_0^\infty \left(\mathbf{K}(\tau) + \mathbf{K}_v(\tau)\right) d\tau$$

For second-order mass-damper-spring systems we choose to approximate Cummins equation at the natural frequency:

$$\omega_0 = \sqrt{\frac{C_{ii}}{\{\mathbf{M}_{RB}\}_{ii} + A_{ii}(\omega_0)}}$$

The natural frequency could also be calculated iteratively by solving:

$$\omega_0^2 \left(\{\mathbf{M}_{RB}\}_{ii} + A_{ii}(\omega_0)\right) - C_{ii} = 0 \qquad (24)$$

From (8)–(9) with $\omega = \omega_0$ we obtain:

$$\mathbf{M}_A^{3,4,5} = \lim_{\omega \to \omega_0} \mathbf{A}^{3,4,5}(\omega) = \mathbf{A}(\infty) - \frac{1}{\omega_0} \int_0^\infty \mathbf{K}(\tau) \sin(\omega_o \tau)\, d\tau$$

$$\mathbf{D}^{3,4,5} = \lim_{\omega \to \omega_0} (\mathbf{B}^{3,4,5}(\omega) + \mathbf{B}_v^{3,4,5}(\omega)) = \int_0^\infty \left(\mathbf{K}(\tau) + \mathbf{K}_v(\tau)\right) \cos(\omega_0 \tau)\, d\tau$$

We will now show how these formulae can be used to derive a frequency-independent maneuvering model in 6 DOF. We will consider one equation for each case to illustrate the relationship between the hydrodynamic derivatives in the manoeuvring equations and the potential coefficients used in the Cummins equation.

3.1.1. Example: Zero-Frequency (LF) Surge Model
The nonlinear maneuvering equation in surge can be written (Fossen 2002):

$$(m - X_{\dot{u}})\dot{u} - X_u u - X_{u|u|} u\,|u| = \tau_1$$

From seakeeping theory, we have that:

$$(m + A_{11}(\infty))\dot{u} + B_{11v} u + \int_0^t K_{11}(\tau)\, u(t-\tau)\, d\tau - X_{u|u|} u\,|u| = \tau_1$$

where we have added the linear skin friction $B_{11v} u$ and a nonlinear viscous term $X_{u|u|} u\,|u|$. The linear hydrodynamic derivatives in surge are related to the Cummins-Ogilvie's equations as follows:

$$-X_{\dot{u}} = A_{11}(\infty) - \int_0^\infty \tau K_{11}(\tau)\, d\tau \qquad (25)$$

$$-X_u = \int_0^\infty K_{11}(\tau)\, d\tau + \beta_1 \qquad (26)$$

These equations come from (22) and (23). In Ross and Fossen (2005), it was shown that, for slow motion, an exponential viscous damping term $B_{11v}(\omega) = \beta_1 e^{-\alpha\omega}$ in the frequency domain can be approximated by the constant term $\beta_1 > 0$ in the time domain. Hence, it is not necessary to evaluate the impulse response function $K_{11v}(t)$ for the viscous part.

3.1.2. Example: Natural-Frequency (NF) Roll Model
From maneuvering theory, the decoupled linear roll equation is written:

$$(I_x - K_{\dot{p}})\,\ddot{\phi} - K_p \dot{\phi} + \overline{BG_z} W \phi = \tau_4 \qquad (27)$$

where BG_z is the vertical distance between the CG and CB. The natural frequency in roll is:

$$\omega_\phi = \sqrt{\frac{\overline{BG_z} W}{I_x - K_{\dot{p}}}} \qquad (28)$$

This frequency can be verified by free decay tests. From seakeeping theory, we have that:

$$(I_x + A_{44}(\infty))\ddot{\phi} + \int_0^t K_{44}(\tau)\, \dot{\phi}(t-\tau)\, d\tau + \int_0^t K_{44v}(\tau)\, \dot{\phi}(t-\tau)\, d\tau + \overline{BG_z} W \phi = \tau_4$$

where $K_{44v}(\tau)$ is the impulse response function due to linearised Ikeda roll-damping $B_{44v}(\omega) = B_{\text{IKEDA}}(\omega)$. This model, as opposed to the maneuvering model, includes the fluid memory effects. The NF model can be derived from Cummins-Ogilvie's equations (8)–(9) with $\omega = \omega_\phi$ as follows:

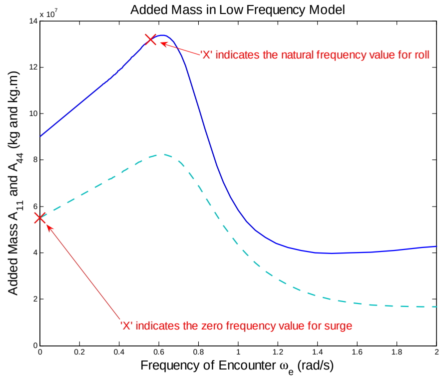

Fig. 1. Added Mass for Surge (broken line) and Roll (full line).

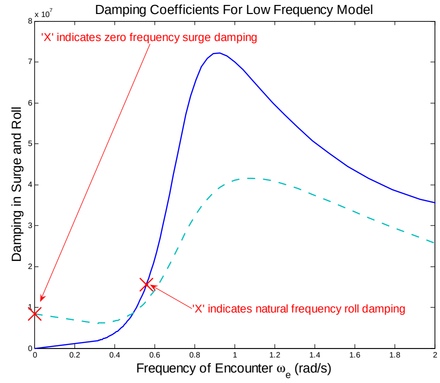

Fig. 2. Damping Coefficients for Surge (broken line) and Roll (full line).

$$-K_{\dot{p}} = A_{44}(\infty) - \frac{1}{\omega_\phi}\int_0^\infty K_{44}(\tau)\sin(\omega_\phi \tau)d\tau \quad (29)$$

$$-K_p = \int_0^\infty \left(K_{44}(\tau) + K_{44v}(\tau)\right)\cos(\omega_\phi \tau)d\tau \quad (30)$$

where the impulse response functions are given by:

$$K_{44}(t) = \frac{2}{\pi}\int_0^\infty B_{44}(\omega)\cos(\omega t)d\omega$$
$$K_{44v}(t) = \frac{2}{\pi}\int_0^\infty B_{44v}(\omega)\cos(\omega t)d\omega$$

These expressions come from equations (8) and (9).

4. NUMERICAL EXAMPLES

Figures 1 and 2 show added mass and damping as functions of frequency for both surge and roll. The cross shows the value that ought to be used in a frequency invariant model. Note that the surge and roll coefficients now come from different frequencies. The surge values are taken at zero frequency, and the roll coefficients are taken at the natural frequency, derived using Equation (24). These values differ significantly from the conventional hydrodynamic derivatives.

5. CONCLUSION

A more suitable approximation of the hydrodynamic equations was presented. It has the same properties of the hydrodynamic derivatives in surge, sway and yaw, but differs significantly in roll, pitch and heave. This reappraisal of manouevring models is a more sound approach to the problem, and serves to improve the quality of simulation models.

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