MATHEMATICAL MODELS FOR MODEL-BASED CONTROL IN OFFSHORE PIPELAY OPERATIONS

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ABSTRACT

This paper considers mathematical models for model-based controller design in offshore pipelay operations. Three classes of models for control design are discussed, real-world models suitable for controller design verification, controller and observer models which are used on-line in the control system implementation. The control application place requirements on the model with respect to the computational time, dynamic behavior, stability and accuracy. Models such as the beam model, two catenary models, as well as general finite element (FE) models obtained from computer programs were not able to meet all of the requirements, and two recent dynamic models designed for control are presented, which bridge the gap between the simple analytical and more complex FE models. For completeness, modeling of the pipelay vessel, stinger and roller interaction, soil and seabed interaction and environmental loads are discussed.

INTRODUCTION

Offshore pipelines are vital in the offshore oil and gas industry and provides safe and reliable transportation of oil and gas at every stage of the process from well to consumer. The construction of large offshore pipelines can only be implemented by a limited number of companies, e.g., [1, 2, 3], that possesses the right equipment and know-how. Pipelines are installed from purpose-built pipelay vessels designed and equipped to handle the construction and lay operation. *Pipelaying* encompass the installation methods where the pipe string is welded together from pipe joints onboard a pipelay vessel as it is installed on the seabed. During the operation the pipe string is freely suspended in the water from the vessel and down to a touchdown point at the seabed, and it can easily be kinked or buckled if the vessel is allowed to drift or rotate. More detailed overviews on installation methods are found in these recently published books: [4, 5, 6, 7].

To ensure the integrity of the pipe, the vessel position and heading must be controlled. Deepwater pipelay vessels rely on *dynamic positioning* (DP) systems, where the vessel position and heading are controlled exclusively by thrusters [8]. The DP system is a *motion control* system that changes the vessel position and heading to a desired reference value. In practice, the DP system is operated by a human operator that provides the position reference based on experience and know-how. Replacing the human operator with a *guidance* system will remove the heuristics and can potentially increase the precision, speed, safety and operational frame for the pipelay operation. For an introduction to guidance laws for marine vessels, see [9] and the many references therein. The background and motivation for guidance in the pipelay operation is given in [10], which this paper is accompanying.

Automation of the pipelay operation rely on suitable mathematical models for both control design and verification of the different control designs. The main objective of this paper is to address the mathematical models required in a model-based control design for the pipelay operation, and in particular models for the pipe. An extensive toolbox of pipe models used for design and simulation of pipelines already exists, see Figure 1. The

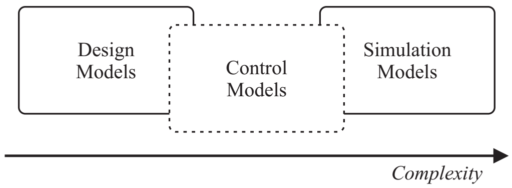

Figure 1. FITTING CONTROL MODELS IN BETWEEN DESIGN AND SIMULATION MODELS.

pipe models used in pipeline design are mainly static models for computing the stress and strains properties of equilibrium configurations, while the simulation models are dynamic or quasi-static models used for operability analysis. *Control models* represents an additional category to the established *design* and *simulation* categories, and in this paper we will define the properties required for a model in this category, and also consider models from the design and simulation categories with respect to these requirements. Apriori knowledge of the control model requirements enables the design of control models directly, as will be shown by an examples.

The exposition of this paper is deliberately kept at a basic level to make it accessible for a wide audience. Details and proofs are found in the references.

INSTALLATION LOADS

The pipe string extending from the stinger to the touchdown point at the seabed is elastic, and its configuration, and thus the position of the touchdown point, is governed by its end point loads and loads along the pipe string. The pipe configuration is the complete specification of the location and orientation of every point of the pipe string, where the upper curve is the so-called *overbend*, and the lower curve is the *sagbend* [11, 12, 13], see Figure 2. The objective in the pipelay operation is to position the pipe touchdown point along a pre-defined path on the seabed by active control of the vessel position and the pay-out speed, while ensuring the structural integrity of the pipe at all times during the operation. The pipelay operation is governed by the three parameters; the *departure angle* from the stinger, *the radius* of the stinger and *the lay tension*, which is the one actively controlled during the pipelay operation.

The level of model complexity is important as models with too much complexity will waste resources, complicate verification and compromise reliability, while too little complexity decreases the model functionality and important behavior is lost. To derive a complete system model for the pipelay operation the following should be included:

1. The pipe dynamics
2. The pipelay vessel dynamics
3. The stinger and roller interaction
4. The soil at the seabed interaction

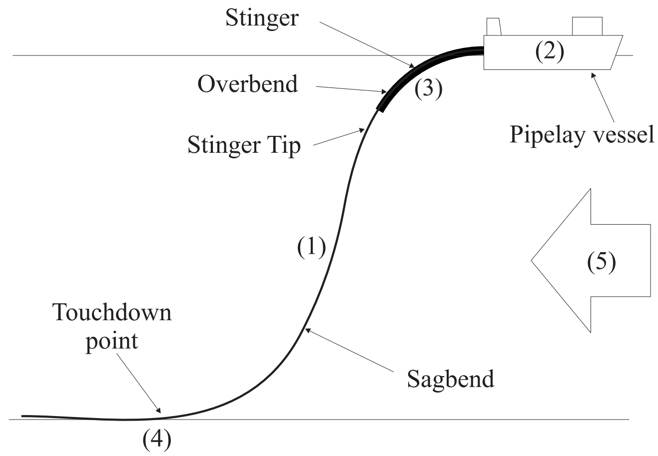

Figure 2. S-LAY PIPE LOADS.

5. The environmental loads.

For motion in water, hydrostatic effects of buoyancy, and hydrodynamic effects of added mass and drag must be considered. The pipelay operation is a low speed application when that the effect of added mass can be neglected. This is particularly valid for the pipe string which has a relatively small displacement. The installation method determines the pipe configuration. For the installation method *S-lay*, the pipe follows the curvature of the stinger in the overbend, but for the *J-lay* method, the pipe is installed near vertically and thus the stinger effects can be neglected. However, the lay method does not affect the configuration in the sagbend, as the equilibrium configuration in the sagbend is load controlled due to no physical boundaries for the deformations of the pipe string. The environmental forces are considered to be wind, waves and current.

MODEL CLASSES

The models used for automatic control are classified in three groups, distinguished by the complexity and number of differential equations needed, see Figure 3. *The real-world model* is the most accurate description of the system, and should be able to reconstruct the time response of the real system. *The controller model* is a reduced order or simplified model of the real-world model used for the implementation of a feedback control system. Model based controllers use a dynamic model to generate the feedforward and feedback signals. *The observer model* is like the control model, a reduced order or simplified real-world model, but will in general be different from the control model, as it is designed to capture the additional dynamics associated with the sensors system. The controller and observer models are used online in the control system implementation. The real-world model is used for validation of different controller schemes in computer simulations that does not impose real-time performance require-

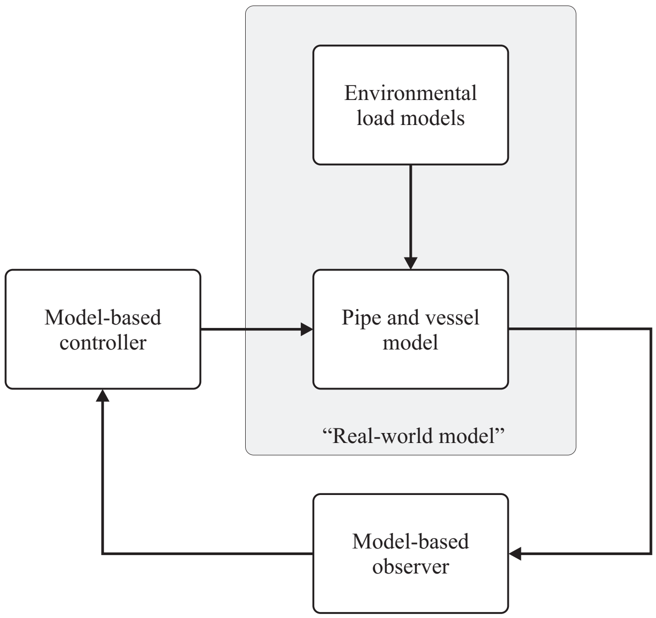

Figure 3. MODEL CLASSES

ments. Validation through simulation is considerably cheaper than sea trials, and there are virtually no limitations to possible test scenarios.

MODEL REQUIREMENTS

Albert Einstein has been attributed the statement: *'Everything should be made as simple as possible, but no simpler'*, which applies well to control and observer models, which should be as simple as possible, while still capture the main system dynamics. The following requirements must be satisfied by the control and observer models implemented in a control system:

1. *Real-time computation* - The model must be simple enough to compute in real-time.
2. *Dynamic* - The model must capture the main dynamics of the pipe.
3. *Stability* - The model must be stable, preferably asymptotically stable, in order to ensure stability for closed-loop control systems.
4. *Accuracy* - The computation must be sufficiently accurate for the application. Integral action in feedback controllers can compensate for offsets.

The accuracy requirement for the real-world model is stricter than that of the controller and observer models, while the real-time computation requirement is relaxed. The requirements on dynamics and stability however, are the same.

STATIC STRESS MODELS

Historically, pipelaying started in shallow waters from open flat-bottomed barges. The pipe curvature can be computed using elastic rod theory in a small strain, large displacement formulation, neglecting axial and torque deformations to analyze the sagbend stresses. This section deals with pipeline models that were used for pipelay operations in the 60s and 70s, namely the beam model, the natural catenary model and the stiffened catenary model.

The Beam Model

The pipe is in its simplest version, a uniform beam with relatively small bending stiffness, loaded only by its own weight. The analytical static equation of equilibrium, including flexural effects and a horizontal tension is well known in structural mechanics, and it is given by the partial differential beam equation:

$$EI\frac{d}{ds}\left(\sec\theta\frac{d^2\theta}{ds^2}\right) - H\sec^2\theta\frac{d\theta}{ds} - mg = 0 \tag{1}$$

where

s	- coordinate along the beam
$d\theta/ds$	- curvature at s
EI	- pipe bending stiffness
H	- horizontal bottom tension
m	- pipe unit mass
g	- standard gravity constant

This equation is also known as the *nonlinear bending equation* and is valid for both deep and shallow waters and small and large deflections [14]. Equation (1) is of second order, with an unknown free pipe length and bottom reaction, so effectively the problem is of fourth order [15]. For this problem no exact solutions are known, and approximations must be considered either by numerical methods, or by equation simplification. Numerical approaches were studied for a beam with small deflections in [16], and a nonlinear method was studied in [17]. If the flexural rigidity vanishes, an exact analytical solutions can be obtained for (1), known as the *natural catenary*. If the pipe weight vanishes too, the equation becomes equivalent to the nonlinear pendulum equation.

The Natural Catenary

The natural catenary is the classic nonlinear solution of the static deflection curve for a string loaded by its own weight, obtain by canceling the flexural term, $EI = 0$, of (1), see Figure 4. The axial tension T at the pipe tip has a vertical and horizontal component, where the vertical component is a function of the

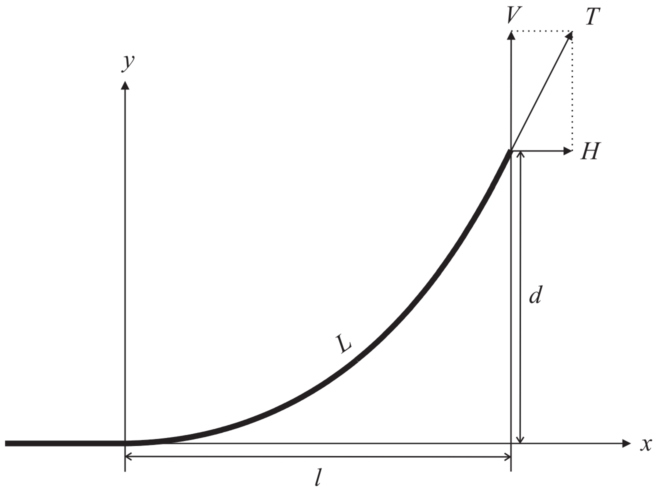

Figure 4. THE NATURAL CATENARY.

water depth d and the unit weight, such that

$$T = H + mgd, \tag{2}$$

and the geometry of the catenary is found in in Cartesian coordinates (x, y) as

$$y = \frac{H}{w}\left(\cosh\left(\frac{wx}{H}\right) - 1\right). \tag{3}$$

When the axial tension dominates the bending stiffness the natural catenary is a good approximation of the sagbend with exception for the boundary regions at both ends. The catenary model has many favorable advantages [18]:

- The extreme conceptual simplicity,
- the solution can be written with simple formulas,
- the results are reliable far away from the ends, and
- it is a good starting point for more refined methods.

The main drawbacks are that it is inaccurate close to the pipe ends, and that the bending moment is discontinues at the touchdown point. The catenary is extensively treated in literature on cable mechanics, such as [19,20] and the references therein. The study of the catenary for pipelay operations benefits from the extensive study on risers, e.g., [21,22].

The tension from the pipe on a pipelay vessel can be considered a nonlinear spring with a horizontal and vertical component [23]. In practice, the vertical component is passively compensated for by the restoring forces on the pipelay vessel, while the DP system must actively control the horizontal component, which is given by

$$l\,(T) = d\left(1 - \sqrt{2\delta - 1} + \ln\left(\gamma + \sqrt{\gamma^2 + 1}\right)^{\delta}\right) \tag{4}$$

where

$$\delta = \frac{T}{mgd} + 1 \quad \text{and} \quad \gamma = \frac{\sqrt{2\delta + 1}}{\delta}. \tag{5}$$

Let the touchdown distance l be the displacement of (4). The value of H can then be found by iteration, or approximated by a polynomial for a fixed water depth d.

The Stiffened Catenary

Improvements were suggested to mend the inaccuracy of the pipe end points by [16], however in 1967, Plunkett [24] suggested the Stiffened Catenary model to find an analytical approximation of the pipe laying problem by introducing an asymptotic expansion to the catenary equation. In the decade to follow, the method was given much attention, and applied to pipelay applications in [25,26,27,28].

Both catenary models meet the real-time requirement for control and observer models, but lacks the dynamic behavior of the pipe string. The methods are best used to compute an approximation of the lay-tension and curvature, or as initial condition for more advanced models. The performance of control systems based on these models will be best at zero speed, and decrease with increasing speed.

NUMERICAL METHODS

When pipelay operations moved into deeper waters, the dynamics of the pipe string became more significant, and with the availability of computers, attention was shifted towards using numerical methods for its compution. There is a plethora of academic papers published in the literature dealing with the global analysis of flexible pipes and risers, which are similar structures, as reviewed by [29] and [30]. The three main numerical methods used for riser and pipeline analysis are

1. Finite-Element Method (FEM),
2. Finite-Difference Method (FDM),
3. Lump-Mass Method (LMM).

Each of these methods can be applied at any conditions of water depths, forces and pipe parameters. References to publications on all the methods are found in [31]. Among the three methods, FEM is perhaps the most widely adopted technique due to its versatility in handling complex flexible pipe profile and boundary conditions.

For industrial applications, the dynamic analysis is mainly done by finite element (FE) based computer tools with different levels of complexity. Some are commercially available, others are for in-house use only. A few examples are: OFFPIPE, RIFLEX, SIMLA, ABAQUS, ORCAFLEX, ANSYS and FLEXCOM-3D. Obtaining the equations of motions for real world systems are often very difficult, and a common approach in these computer tools is to obtain a set of equations by combining discrete beam or bar elements into a connected structure, linked by the forces and moments between the elements. Simulations with good accuracy of pipes with inhomogeneous density, and pipes with in-line structures is obtained. See [32] for an example where OFFPIPE has been applied.

These models may be suitable as real-world models for controller scheme verification, but are unfortunately not suitable for control and observer models. The accuracy of the dynamic behavior is impeccable, but the computation is generally slow, and can not meet real-time requirements for on-line computation. Furthermore, the stability properties are very hard to determine. For finite element models that are derived from partial differential equations (PDE) using the finite element method, a stability analysis of the PDE and the discretization method can be done, but when the model is constructed from discrete elements, there is no PDE to perform the stability analysis on. Following [33], controllers based on such finite dimensional models can become unstable when connected to infinite-dimensional systems, due to the un-modeled modes in the system, the so-called *spillover*.

Abandoning the finite element models for control applications all together is however a hasty decision, as the stability properties of finite element pipe models derived from PDEs can be found. Computation time is still an issue, however reducing the number of discrete elements and using modern computer technology may still yield adequate results.

In the following, the PDE for a nonlinear pipe model in three-dimensions, capable of undergoing finite extension, shearing, twist and bending is given. The details, discretization and stability proof is found in [23]. The dynamics of the nonlinear pipe in a spatial frame e fixed at the touchdown point be

$$m_P \ddot{\boldsymbol{\varphi}} = \partial_S \mathbf{n} - \mathbf{f}_g - \mathbf{R}_t^e \mathbf{f}_D \tag{6}$$

$$\mathbf{I}_\rho \dot{\mathbf{w}} + \mathbf{w} \times \mathbf{I}_\rho \mathbf{w} = \partial_S \mathbf{m} + (\partial_S \boldsymbol{\varphi}) \times \mathbf{n} - \mathbf{D}_R \mathbf{w} \tag{7}$$

where

m_P	- mass per unit length
$\mathbf{R}_t^e \mathbf{f}_D$	- transversal hydrodynamic damping matrix
$\mathbf{n}$	- resultant internal force vector
$\mathbf{f}_g$	- restoring forces vector (gravitation/buoyancy)
$\mathbf{I}_\rho$	- mass moment of inertia matrix
$\mathbf{D}_R$	- constant damping matrix of rotation
$\mathbf{m}$	- resultant internal torque vector.

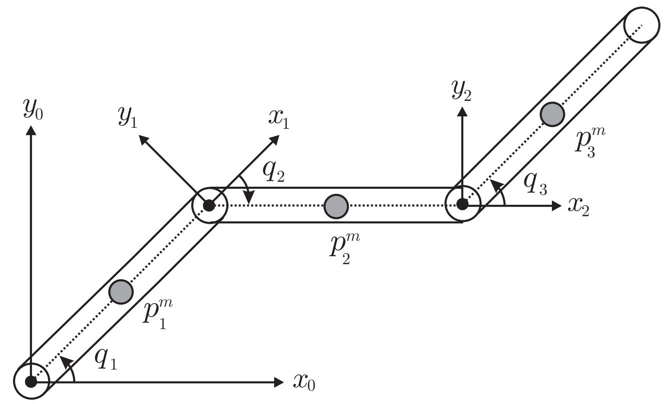

Figure 5. ROBOTIC PIPE MODEL LINKS.

The configuration of the pipe is given by a smooth curve $\boldsymbol{\varphi} : [0, L] \rightarrow \mathbb{R}^3$ through the *line of centroids*, where the position of any point on this line is given by $\boldsymbol{\varphi}(S)$ and the orientation of the cross-section at $\boldsymbol{\varphi}(S)$ is given by the rotation matrix $\mathbf{R}_t^e(S)$, where $t(S)$ is a body fixed frame with origin at S. Thus the pipe configuration is completely defined by specifying the evolution of the position vector of the line of centroids $\boldsymbol{\varphi}(S, t)$ and the orthogonal rotation matrix $\mathbf{R}_t^e(S, t)$ along the material variable S. The vector $\mathbf{w}$ denotes the angular velocity, ∂_S denotes the material derivative, and a superposed dot denote the time derivative. The vessel equation of motions and the seabed are taken as the upper and lower boundary conditions respectively [34].

A ROBOTIC PIPE MODEL

It seems that pipe models used for design and simulation are best suited for computing initial estimates or for control design verification, and not suitable for control applications. This has motivated a nonlinear dynamic modeling approach for offshore pipelaying for control applications based on robotics [35, 36]. Following [37], the pipe is modeled as hyper redundant planar robot manipulator of i links of length l_i with point masses p_i^m, and bending stiffness represented by torsional springs in the joints, see Figure 5. The model is linked to the pipelay vessel model through tension. For a robot manipulator, the generalized coordinates in the joint space are

$$\mathbf{q} = \begin{bmatrix} q_1 \; q_2 \; \ldots \; q_n \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^n, \tag{8}$$

and the equation of motion of the pipe in the vertical plane is developed using a Lagrangian formulation in the joint space given by

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{H}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{f}(\mathbf{q}) + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau}, \tag{9}$$

where

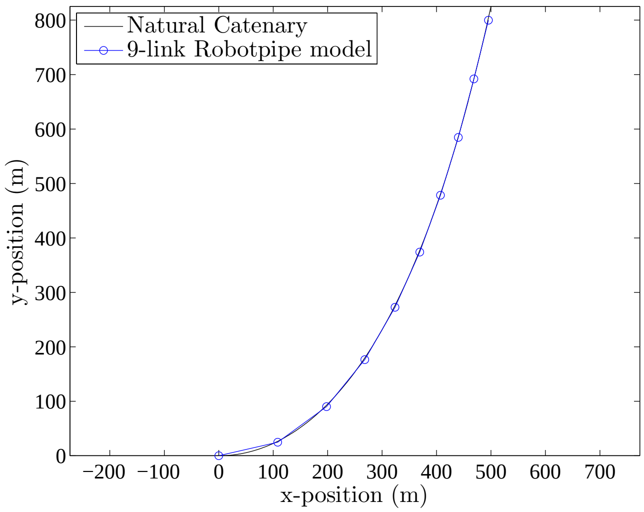

Figure 6. THE ROBOT MODEL OF NINE LINKS PLOTTED ON TOP OF THE NATURAL CATENARY.

$\mathbf{M}(\mathbf{q})$	- system inertia matrix
$\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})$	- Coriolis-centripetal matrix
$\mathbf{H}(\mathbf{q},\dot{\mathbf{q}})$	- hydrodynamic damping matrix
$\mathbf{f}(\mathbf{q})$	- vector of spring forces in the pipe joints
$\mathbf{g}(\mathbf{q})$	- vector of gravitational/buoyancy forces
$\boldsymbol{\tau}$	- vector of pipelay vessel effects

where $\mathbf{M},\mathbf{C},\mathbf{H} \in \mathbb{R}^{n\times n}$ and $\mathbf{g},\mathbf{f},\boldsymbol{\tau} \in \mathbb{R}^{n}$. The form of (9) is well known in control engineering, and several methods for stability analysis are available. The static configuration of the robotic pipe with no bending stiffness is plotted on top of the natural catenary of in Figure 6, where the two configurations are seen to overlap well. This system is shown to be stable and applied in a DP application in [37].

BOUNDARY CONDITIONS AND ENVIRONMENT

To derive a complete model for offshore pipelay operations, additional loads in the system should be considered as earlier mentioned. This section covers the pipelay vessel dynamics, stinger and roller interaction, seabed interactions and environmental loads.

Pipelay Vessel Dynamics

Following [38], the vessel is a rigid body subject to hydrodynamic forces, and written on vectorial form inspired by the robot model [36] as

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta})\boldsymbol{\nu} \tag{10}$$

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \mathbf{g}(\boldsymbol{\eta}) = \boldsymbol{\tau} + \boldsymbol{\chi} + \mathbf{w} \tag{11}$$

where

$\mathbf{M}$	- system inertia matrix
$\mathbf{C}(\boldsymbol{\nu})$	- Coriolis-centripetal matrix
$\mathbf{D}(\boldsymbol{\nu})$	- damping matrix
$\mathbf{g}(\boldsymbol{\eta})$	- vector of gravitational/buoyancy forces
$\boldsymbol{\tau}$	- vector of control inputs
$\boldsymbol{\chi}$	- vector of pipe tension
$\mathbf{w}$	- vector of environmental loads

and $\boldsymbol{\eta} = [\mathbf{p},\boldsymbol{\Theta}]^{\mathrm{T}} \in \mathbb{R}^3 \times \mathcal{S}^3$ denotes the generalized position vector where the position $\mathbf{p}$ is decomposed in an inertial reference frame, and $\boldsymbol{\Theta}$ is a vector of Euler angles. The linear and angular velocity in the body fixed frame is denoted by $\boldsymbol{\nu} \in \mathbb{R}^6$, in accordance with [39]. The system properties represent physical properties of the system, which may be exploited when designing controllers and observers, and the system properties of symmetry, skew-symmetry and positiveness of matrices simplifies the stability analysis.

In recent years there has been a significant drive to develop time-domain models for simulation and control system design based on data obtained from seakeeping programs such as ShipX and WAMIT. These programs use potential theory to compute the potential coefficients and the existing wave loads for a given vessel design, and a potential theory formulation for a surface vessel suited for dynamic positioning and low speed maneuvering is developed [40,41].

Stinger Interaction

In S-lay mode, the stinger supports the overbend of the pipe on rollers spaced out along the stinger. Older stingers are rigid, whereas modern ones are articulated, involving several segments connected by hinges. The roller contacts are monolateral and the pipe cannot reach a radius of curvature higher than the one imposed by the stinger, and it seems reasonable to say that from the third roller from the tip and up, the pipe is displacement controlled. References to modeling of the impact of the stinger are: [11,42,43,44].

Soil and Seabed Interaction

The seabed represents the lower boundary for the pipe. The pipe route is selected and prepared to be as smooth and flat as possible before the pipelay starts. The seabed soil varies from solid rock and hard clay to soft clay and mud. Seabed intervention work such as trenching and rock dumping is often performed before and after the pipelay. A common approach for modeling the contact forces between the pipe and the soil, used in e.g., [45], is lumped spring and damper pairs that can be tuned to obtain different soil properties. Computations for deep water J-lay on rigid and Winkler soil is done in [18]. See also [46].

Environmental Loads

The environmental loads are considered to be *wind*, *waves* and *ocean currents*, which must be considered for any offshore control system. The pipelay vessel is subject to wind, waves and current loads, whereas the pipelay vessel is mainly subject to the current. The vessel is not allowed to fully weathervane to reduce the impact of these loads during the pipelay operation.

For the pipelay vessel, the wind and wave loads $\mathbf{w}$ in (11) follows the *principle of superposition*, such that

$$\mathbf{w} = \boldsymbol{\tau}_{wind} + \boldsymbol{\tau}_{wave_1} + \boldsymbol{\tau}_{wave_2}. \tag{12}$$

The effects of the wind $\boldsymbol{\tau}_{wind}$ can be compensated for by the DP system based on wind coefficients, wind speed and directional measurements [38]. The wave loads are wind induced and can be separated in 1st-order waves $\boldsymbol{\tau}_{wave_1}$, which is a wave frequency motion with zero mean, and 2nd-order waves $\boldsymbol{\tau}_{wave_2}$, which is a non-zero slowly-varying drift force [47]. Wave excitation forces on the pipe can be neglected since they only affect the pipe near to the surface, typically down to 20 meters depth. The current $\boldsymbol{\nu}_c$ is assumed to be slowly-varying, such that $\dot{\boldsymbol{\nu}}_c \approx 0$, and influences both the vessel and the pipe. The current load is integrated by substituting the velocity $\boldsymbol{\nu}$ by the relative velocity $\boldsymbol{\nu}_r = \boldsymbol{\nu} - \boldsymbol{\nu}_c$ in appropriate places in (11) and the pipe models respectively. The pipe will experience different directions and speeds for the current at different water depths due to current profiles, which must be accounted for [48].

CONCLUSIONS

This paper has considers mathematical models for model-based controller design, and in particular guidance systems, in offshore pipelay operations. The category of control models has been added to the categories for design and simulation models already defined. Models in the latter categories will in most cases not be able to meet the requirements of control applications, thus a new models must be provided, such as the robotic pipe model and the finite strain PDE pipe formulation. Models for the pipelay vessel, stinger and roller interaction, soil and seabed interaction and environmental loads are presented for completeness.

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